Optimal Resource-Constrained Periodic Scheduling

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1 Introduction

Periodic scheduling problems have **applications** in various domains, such as: **manufacturing**, **logistics**, **embedded computing**.

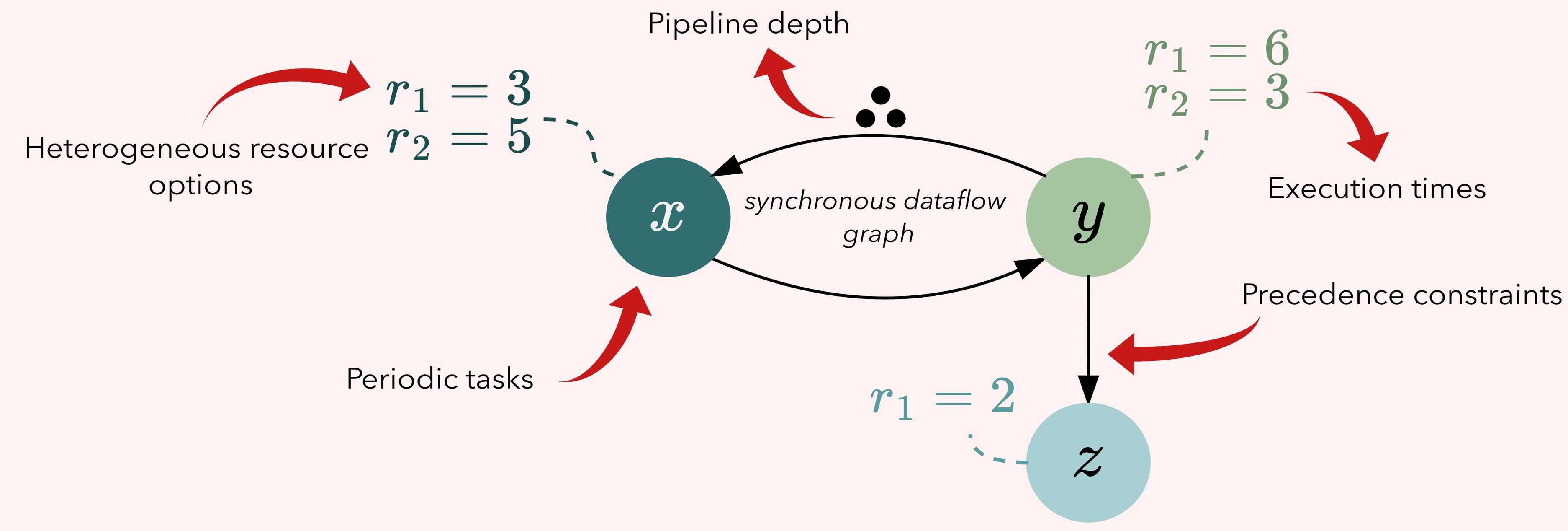
These periodic scheduling problems involve **systems** that process **tasks** that **repeat indefinitely** with a specified period.

Often these systems are constrained by **limited shared resources**.

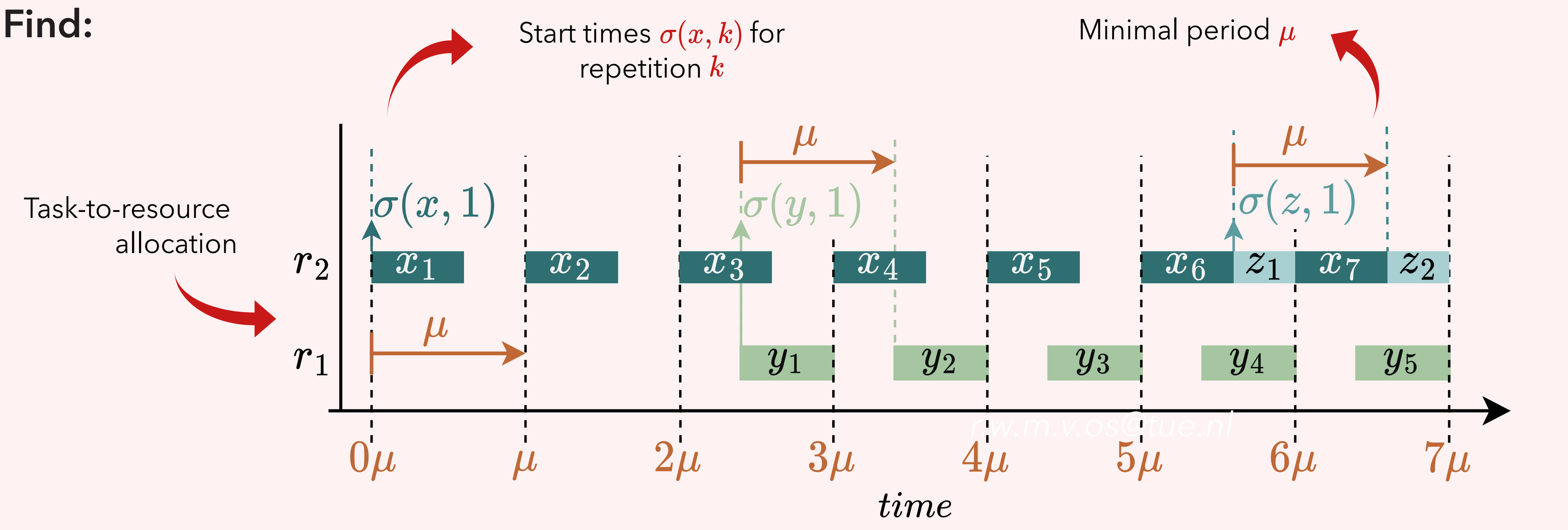
The goal is to find the **optimal resource allocations** and a **strictly periodic schedule** with a **minimal period.**

2 Problem Statement

Given:



Find:



3 Modeling

Resource constraints (easy)

Ensures each task is allocated to a resource.

$$a : T \rightarrow R$$

Complete function allocating all tasks from the set T to resources from the set R

Precedence constraints (easy)

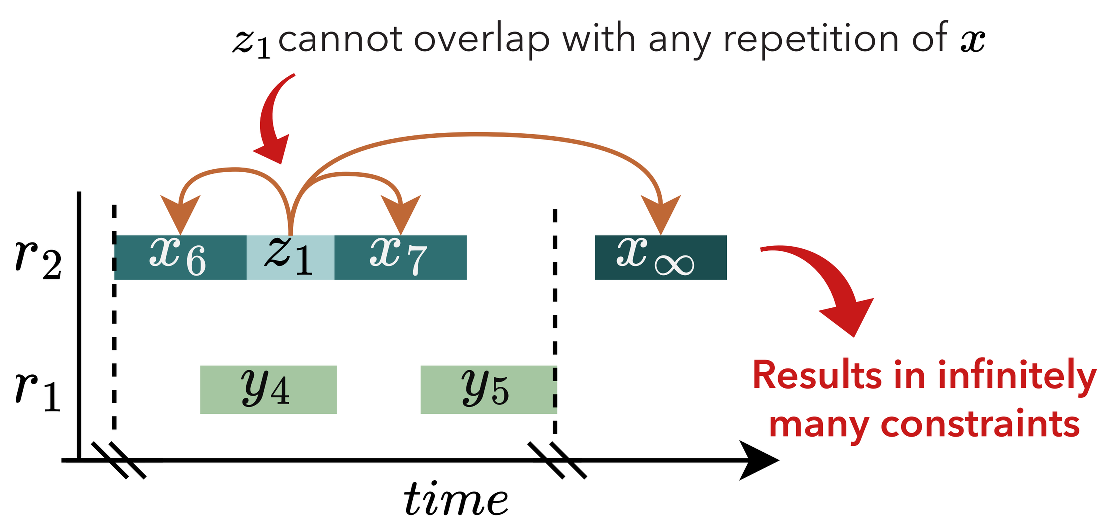
Ensures the correct task ordering. E.g., for tasks z and y:

$$\sigma(z,k) \geq \sigma(y,k) + e(y, a(y))$$

Task duration on the allocated resource

No-overlap constraints (hard)

Ensures no overlap between tasks on the same resource.



4 Our Solution

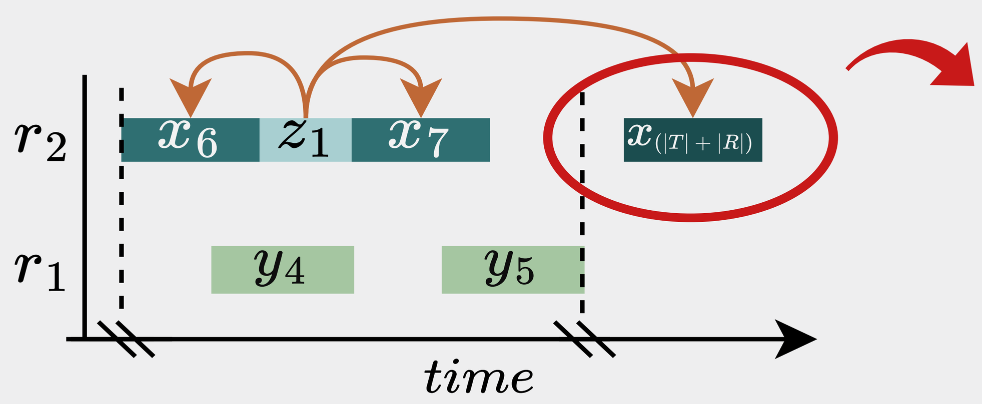
Bounding the repetitions

We proved: If there exists a schedule, there also exists an optimal schedule where for all $t \in T$:

$$\sigma(t,1) \leq (|T| + |R|) \cdot \mu$$

where $|T|$ and $|R|$ represent the number of tasks and resources, respectively.

No-overlap constraints



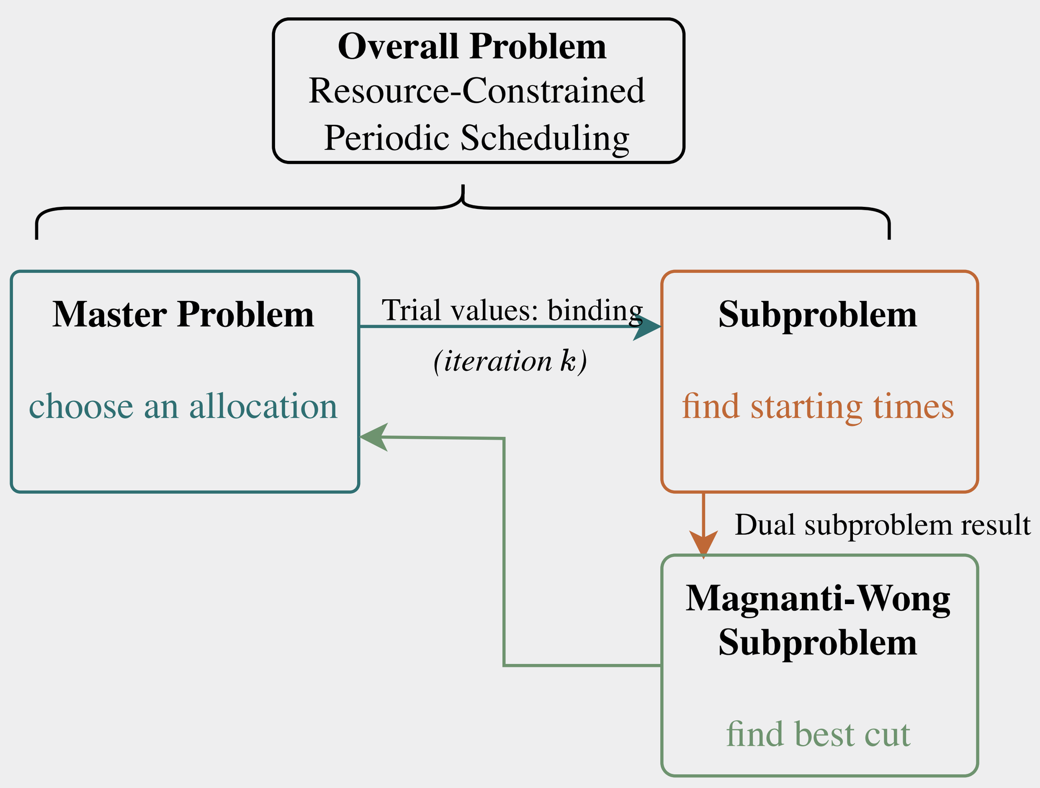
- Every first task instance is scheduled before $(|T| + |R|) \cdot \mu$
- After the first instance **all tasks repeat periodically**
- **No-overlap satisfaction** can be checked with the first $|T| + |R|$ iterations
- Only **finitely many constraints**

5 Models

Original Models

Two **Mixed Integer Linear Programming (MILP)** models were created to find the optimal schedule for a given problem instance:

1. A **monoltithic** MILP model using our **bounding technique**.
2. A **Benders decomposition** applying the **Magnanti and Wong Acceleration** techniques



Extended State-of-the-Art Models

Two models were created **extending the state-of-the-art** MILP models from Quinton (2020) by applying our bounding technique.

6 Experiments

Comparison

We evaluate our models and the extended state-of-the-art models against the state-of-the-art models by Quinton (2020):

Our Original Models

1. **Our Monolithic** model
2. **Our Benders** decomposition

State-of-the-art models:

3. **Quinton Monoltihic** model
4. **Quinton Benders** decomposition

Our extened state-of-the-art models

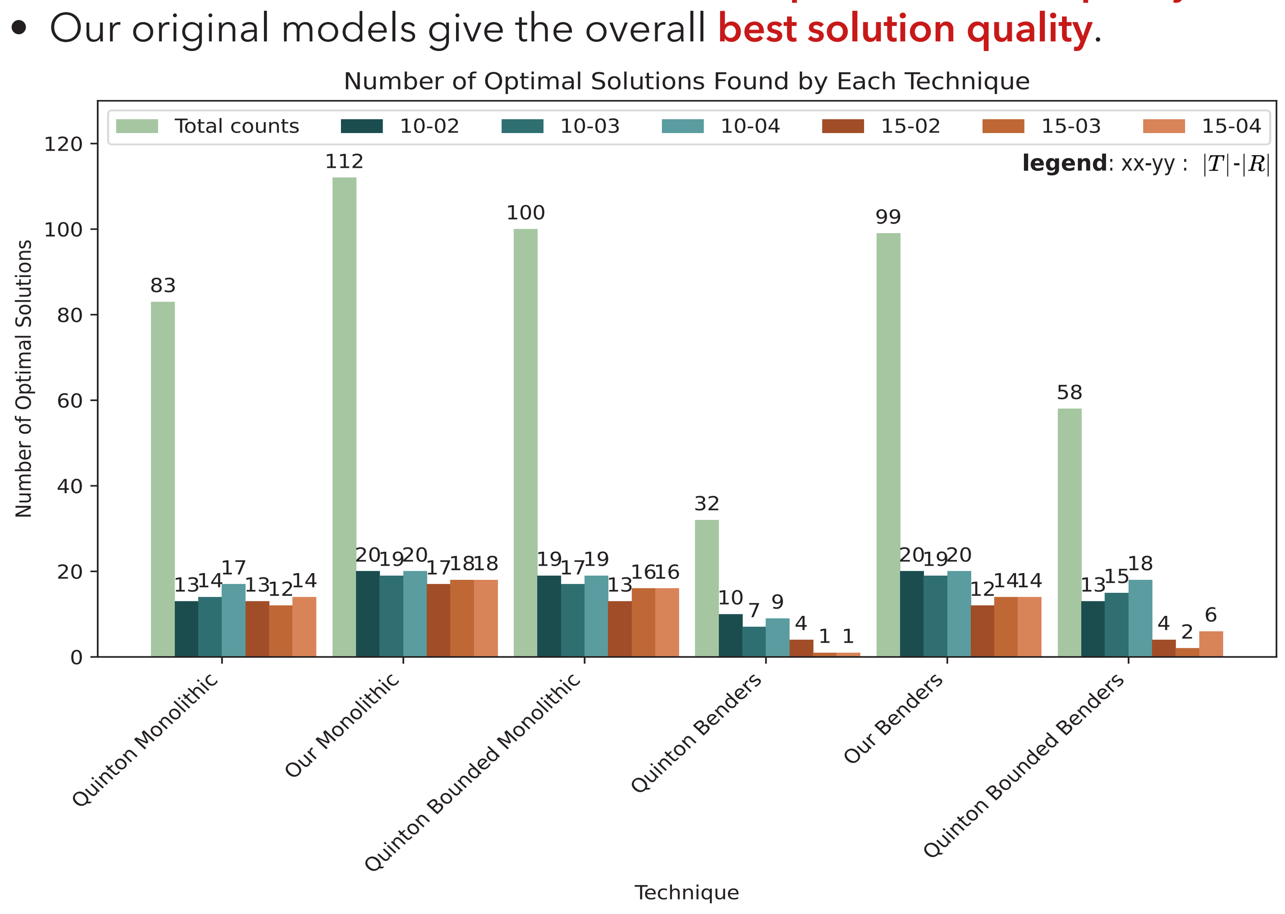
5. **Quinton Bounded Monolithic** model
6. **Quinton Bounded Benders** decomposition

Benchmark

A benchmark of **120 problem instances** was created to evaluate the performance of the models. The benchmark consist of **6 subsets of problem instances** with 10-15 tasks and 2-4 resources.

Solution Quality

- Our extended state-of-the-art models **improve solution quality**.
- Our original models give the overall **best solution quality**.



Solve Time

- Our models are competitive to the state-of-the-art in solve time.
- In most cases our models also **outperform state-of-the-art in solve time.**

Reference

Quinton, F., Hamaz, I., Houssin, L., 2020. A mixed integer linear programming modelling for the flexible cyclic jobshop problem. Ann Oper Res 285, 335-352. https://doi.org/10.1007/s10479-019-03387-9

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