(a) $(n-3)^2 = (\sqrt{3n+1})^2$

$(n-3)^2 = 3n+1$

$n^2 - 6n + 9 = 3n + 1$

$n^2 - 9n + 8 = 0$

$(n-8)(n-1) = 0$

$n = 8\ ,\ n = 1$

Sub back in to verify: $n = 8 \rightarrow 5 = \sqrt{25}$
$n = 1 \rightarrow\ -2 \neq \sqrt{4}$

$n = 8$

(b) $\frac{4}{2t+1}\left(\frac{12t}{12t}\right) - \frac{7}{12t}\left(\frac{2t+1}{2t+1}\right)$

$\frac{48t}{12t(2t+1)} - \frac{14t+7}{12t(2t+1)}$

$\frac{48t-14t-7}{(12t)(2t+1)}$

(c) $x + 2y - 0w = 143$
$0x + y + 3w = -74 \qquad -(2)$

$x + 2y - 0w = 143$
$-0x - 2y - 6w = 148$
$x - 6w = 291$

$x - 6w = 291 \qquad -(4)$
$4x + 5w = 4$

$-4x + 24w = -1164$
$4x + 5w = 4$
$29w = -1160$

$w = -40$

$x - 6(-40) = 291$
$x = 51$

$(51) + 2y = 143$
$y = 46$

(a) Use the -b formula $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = 1\ ,\ b = 12\ ,\ c = 261$

$\frac{-12 \pm \sqrt{12^2-4(1)(261)}}{2(1)}$

$\frac{-12 \pm \sqrt{-900}}{2}$

$\frac{-12+30i}{2}\ or\ \frac{-12-30i}{2}$

$-6 + 15i\ \ or\ \ -6 - 15i$

(b) $r = \sqrt{1^2 + (\sqrt{3})^2} = 2$

$\theta = tan^{-1}\frac{1}{\sqrt{3}} = 60°$

$360 - 60 = 300°$

$r(cos\theta + isin\theta)$

$1 + 3i = 2(cos300 + isin300)$

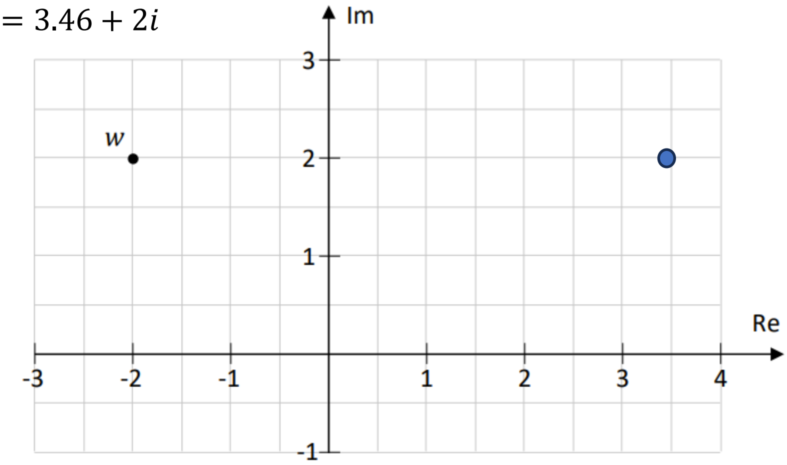
$(1 + 3i)^9 = \left(2(cos300 + isin300)\right)^9$

$512(cos9(300) + isin9(300)))$

$512(-1 + 0i)$

-512

(c) (i) $u = 3.46 + 2i$



(ii) angle of elevation of w (argument): $\frac{3\pi}{4}$

angle of elevation of u: $\frac{\pi}{6}$

$$\frac{3\pi}{4} - \frac{\pi}{6} = \frac{7\pi}{12}$$

(a) $\frac{sin6x}{6} + c$

(b) (i) $f'(x) = 6x^2 - 18x + 5$

$f'(2) = 6(2)^2 - 18(2) + 5 = -7$ (slope of tangent)

$f(2) = 2(2)^3 - 9(2)^2 + 5(2) - 11 = -21$ Point: $(2, -21)$

Equation of the line formula: $y - y_1 = m(x - x_1)$

$y - (-21) = -7(x - 2)$

(ii) Point of inflection: use second derivative = 0

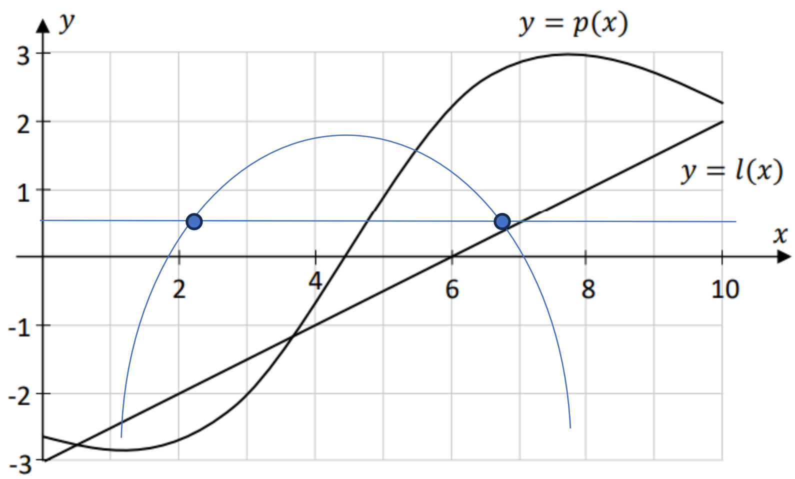
$f'(x) = 6x^2 - 18x + 5$

$f''(x) = 12x - 18 = 0$

$12x = 18$

$x = \frac{3}{2}$

(c)



- Draw the two derivatives: $l'(x)$: straight line at $\frac{1}{2}$
- $p'(x)$: draw curve at the two turning point x-values but on the x-axis
- Where they intersect: $x \approx 2.2 \; and \; x \approx 6.8$

(a) $f(x+h) = (x+h)^2 - 7(x+h) - 10$

$x^2 + 2xh + h^2 - 7x - 7h - 10$

$f(x+h) - f(x) = x^2 + 2xh + h^2 - 7x - 7h - 10 - (x^2 - 7x - 10)$

$f(x+h) - f(x) = 2xh + h^2 - 7h$

$\frac{f(x+h)-f(x)}{h} = \frac{2xh+h^2-7h}{h}$

$2x + h - 7$

h lim 0 → $2x + (0) - 7$

$f'(x) = 2x - 7$

(b) Quotient rule: $\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$u = 6x + 1$, $v = x^4 + 3$

$\frac{du}{dx} = 6$, $\frac{dv}{dx} = 4x^3$

$$g'(x) = \frac{(x^4+3)(6) - (6x+1)(4x^3)}{(x^4+3)^2}$$

$$g'(-2) = \frac{((-2)^4+3)(6) - (6(-2)+1)(4(-2)^3)}{((-2)^4+3)^2} = -\frac{238}{361}$$

(c) Answer: false

Justification: (0, 5) is only a local min. There might be points away from $x = 0$ that are below 5.

(a) $T_2 - T_1 = T_3 - T_2$

$5p - 3 - (2p + 1) = 6p + 7 - (5p - 3)$

$3p - 4 = p + 10$

$2p = 14$

$p = 7$

(b) $\frac{G_{11}}{G_7} = r^4$

$G_{11} \div G_7 = r^4$

$\frac{3}{8} \div 6 = r^4$

$\frac{1}{16} = r^4$

$r = \frac{1}{2}$

(c) (i) $F_1 = 2024x^{2023}$

$F_2 = 2024(2023x^{2022})$

(ii) $2024 - n$

At $n = 2024$, answer is a constant (x^0)

So at $n = 2025$, answer is 0

Answer: $n = 2025$

(a) If $(x-4)$ is a factor, then $x-4=0$ and $x=4$

$h(4)=(4)^2+b(4)-12$

$16+4b-12=0$

$4b=-4$

$b=-1$

(b) (i) $f(1.2)=\ e^{9(1.2)}=49{,}020.8$

4.9×10^4

(ii) $ln\sqrt{x}=3.5$

$e^{3.5}=\sqrt{x}$

$x=(e^{3.5})^2$

$x=e^7$

(iii) $g\big(f(x)\big)=ln\sqrt{e^{9x}}$

$lne^{\frac{9x}{2}}$

$\frac{9x}{2}(lne)$

$\frac{9x}{2}$

$4.5x$

(a) $20\%\ of\ 40{,}000 = 8{,}000$

$40\%\ of\ (54{,}000 - 40{,}000) = 5{,}600$

Total tax: $8{,}000 + 5{,}600 = 13{,}600$

Less tax credit: $13{,}600 - 1{,}775 = 11{,}825$

Net pay: $54{,}000 - 11{,}825 = €42{,}175$

(b) (i) $\frac{1647.75}{1.00279} + \frac{1647.75}{1.00279^2} + \frac{1647.75}{1.00279^3}$

(ii) $\frac{1647.75}{1.00279} + \frac{1647.75}{1.00279^2} + \cdots + \frac{1647.75}{1.00279^{300}}$

$S_n = \frac{a(1-r^n)}{1-r}$

$a = \frac{1647.75}{1.00279}$, $r = \frac{2nd\ term}{1st\ term} = \frac{1647.75}{1.00279} \div \frac{1647.75}{1.00279^2} = \frac{1}{1.00279}$

$S_n = \frac{\frac{1647.75}{1.00279}\left(1-\left(\frac{1}{1.00279}\right)^{300}\right)}{1-\frac{1}{1.00279}} = €334{,}563$ (nearest euro)

(c) (i) $F'(t) = (0.4)(5000e^{0.04t})$

$F'(3.5) = 200e^{0.04(3.5)} = 230.05$

€230

(ii) $\frac{1}{5}\int_0^5 (5000e^{0.04t})\ dx$

$\frac{1}{5}(5000)\left\{_0^5 \frac{e^{0.04t}}{0.04}\right\}$

$1000\left(\frac{e^{0.04(5)}}{0.04} - \frac{e^{0.04(0)}}{0.04}\right)$

€5,535 (nearest euro)

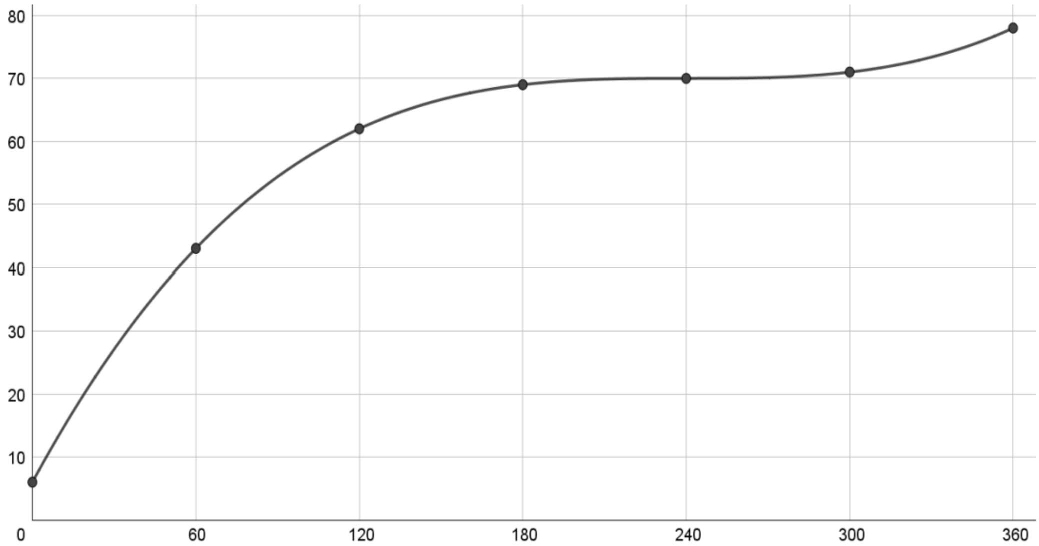
(iii) $e^{0.04} = 1.04081$

4.08%

(a) (i)

x	0	60	120	180	240	300	360
$T(x)$	6	43	62	69	70	71	78

(ii)



(b) Maximum: $21 + 19(1) = 40$

Minimum: $21 - 19(1) = 2$

(c) $C(t) = S(t)$

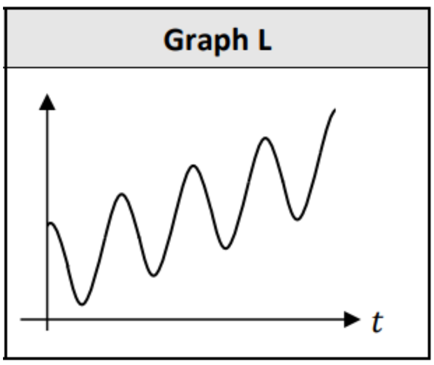
$S(t) = S(t) - 2.4 + 0.03t$

$-2.4 + 0.03t = 0$

$0.03t = 2.4$

$t = 80$ days

(d)



(e) $0.03 - \frac{38\pi}{365}\sin\left(\frac{2\pi t}{365}\right) = 0$

$0.03 = \frac{38\pi}{365}\sin\left(\frac{2\pi t}{365}\right)$

$\frac{0.03(38\pi)}{365} = \sin\left(\frac{2\pi t}{365}\right)$

$sin^{-1}\left(\frac{0.03(38\pi)}{365}\right) = \frac{2\pi t}{365}$

$0.09815 = \frac{2\pi t}{365}$

$t = 5.336 = 5$ days

(a) $\frac{\pi(4)^2}{3}(3(13) - 4) = \frac{560\pi}{3}$

(b) (i) $\frac{\pi y^2}{3}(3(8) - y = 36\pi y)$

$\frac{y^2}{3}(24 - y) = 36y$

$\frac{y}{3}(24 - y) = 36$

(ii) $8y - \frac{y^2}{3} = 36$

$24y - y^2 = 108$

$y^2 - 24y + 108 = 0$

$(y - 6)(y - 18) = 0$

$y = 6 \quad y = 18$

$y = 6$ as less than8

(c) 3 litres $= 3{,}000\ cm^3$

$3{,}000 = \frac{\pi}{12}x^3$

$x^3 = \frac{3{,}000}{\frac{\pi}{12}}$

$x = \sqrt[3]{\frac{12(3{,}000)}{\pi}} = 22.5$ cm

(d) $\frac{dv}{dt} = 450$

$v = \frac{\pi}{12}x^3$

$\frac{dv}{dx} = \frac{\pi}{4}x^2$

$\frac{dx}{dv} = \frac{4}{\pi x^2}$

$\frac{dx}{dv} \times \frac{dv}{dt} = \boldsymbol{\frac{dx}{dt}}$

$\frac{dx}{dt} = \frac{4}{\pi x^2}(450)$

$x = 20 \rightarrow \frac{1{,}800}{\pi(20^2)} = 1.4$ cm/second

(e) $\frac{S}{\pi r} = \sqrt{r^2 + h^2}$

$\frac{S}{\pi^2 r^2}^{2} = r^2 + h^2$

$h^2 = \frac{S^2}{\pi^2 r^2} - r^2$

$h = \sqrt{\frac{S^2}{\pi^2 r^2} - r^2}$

$h = \frac{\sqrt{S^2 - \pi^2 r^4}}{\pi r}$

(a) (i) $W(15) = 0.667(15) + 1.5(15)^2 - 0.025(15)^2 = 263$ mm

(ii) $W'(x) = 0.667 + 3x - 0.075x^2$

(b) $1.1 + 2.73x - 0.078x^2 > 24$

$0.078x^2 - 2.73x + 22.9 < 0$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\frac{-(-2.73)\pm\sqrt{(-2.73)^2-4(0.078)(22.9)}}{2(0.078)}$

$= 21.06 \ or = 13.94$

$14 \leq x \leq 21$

(c) (i) $s = \int_0^1 2x - x^2 \, dx$

$\left\{ {}^1_0 x^2 - \frac{1}{3}x^3 \right\}$

$\left((1)^2 - \frac{1}{3}(1)^3\right) - \left((0)^2 - \frac{1}{3}(0)^3\right) = \frac{2}{3}$

$c = \int_0^1 x^2 \, dx$

$\left\{ {}^1_0 \frac{1}{3}x^3 \right\}$

$\left(\frac{1}{3}1^3\right) - \left(\frac{1}{3}(0)^3\right) = \frac{1}{3}$

Area $= 2(s - c)$

$2\left(\frac{2}{3} - \frac{1}{3}\right) = \frac{2}{3}u^2$

(ii) $k(x) = s(-x)$

So $k(x) = 2(-x) - (-x)^2$

$-2x + x^2$

Therefore: $b = -2 \ , \ c = 0$

(d) Option 1

Price for option 1: $0.9p - r$

Price for option 2: $0.9p - 0.9r$