(a) (i) Mode is 34 therefore $a = 4$

(ii) $69 - 20 = 49$ the range

Therefore $b = 0$, $c = 9$

(iii) Median: $\frac{4d+45}{2} = 43.5$

$4d + 45 = 87$

$4d = 42$

Therefore $4d = 42 \rightarrow d = 2$ (using the key)

(b) Everyone improved but the stronger swimmers improved to a greater extent than the weaker swimmers

(c) $r = 0.9145$

(a) $0(0.3) + 2(0.4) + (x - 10)(0.28) + x(0.02) = 10$

$0.8 + 0.28x - 2.8 + 0.02x = 10$

$0.3x - 2 = 10$

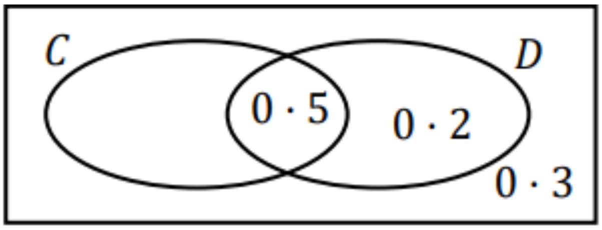
$0.3x = 12$

$x = 40$

(b) $P(A \cap B) = 0$

$P(A \cup B) = 0.5$

(c)



To maximise the set of numbers excluding the intersection of C and D, the intersection minimum has to be 0.5 to include all of C, after that $1 - 0 - 0.5 - 0.2 = 0.3$

(d) If it is raining one is more likely to wear a coat than if it isn't

(a) Area of the triangle ABC: $\frac{1}{2}abSinC \rightarrow \frac{1}{2}(10)(13)sin110 = 61.08$

Area of Parallelogram $ABCD$: $2(61.08) = 122cm^2$ (nearest cm)

(b) Angle will be in the first and fourth quadrants

$cos^{-1}\frac{\sqrt{3}}{2} = 30$

$0 + 30 = 30 \quad and \quad 360 - 30 = 330$

$2x = 30, 330, 390. 690$

$x = 15°, 165°, 195°, 345°$

(c) $\frac{SinA}{a} = \frac{SinB}{b}$

$\frac{Sin(angle\ KLM)}{KM} = \frac{Sin(angleLKM)}{ML}$

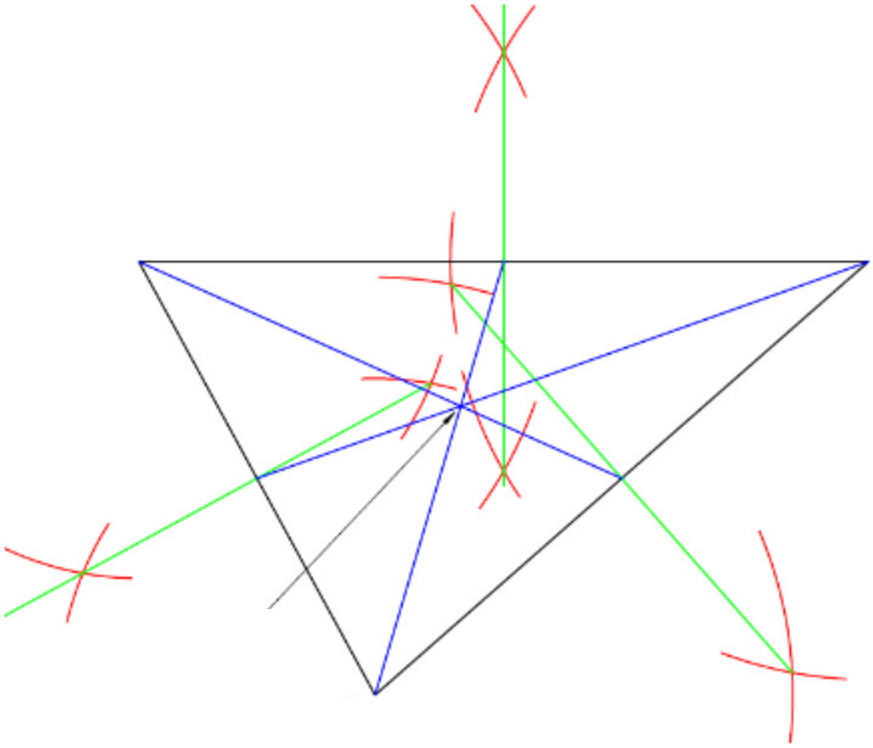
$\frac{Sin25}{15\sqrt{3}} = \frac{Sin\theta}{45}$

$Sin\theta = \frac{45Sin25}{15\sqrt{3}} = 0.73196$

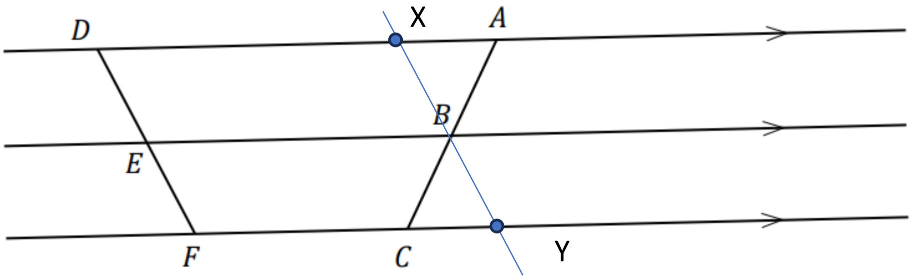
$\theta = sin^{-1}(0.73196) = 47.05°$ in the first and second quadrants

$0 + 47 = 47°\ ,\ 180 - 47 = 133°$

(a)



(b)



$|AB| = |BC|$ given
$Angle\ XAB = Angle\ BCY$ alternate angles
$Angle\ XAB = Angle\ BCY$ Vertically opposite angles
Triangle XBA is similar to Triangle YBC as Angle Side Angle
Therefore $|BX| = |BY|$ as congruency

But $DEBX$ is a parallelogram, as the opposites sides are parallel.
So $|BX| = |DE|$ as opposite sides of a parallelogram

Similarly $|BY| = |EF|$
So $|DE| = |EF|$

(a) (i) Centre: $(-2\,,3)$

Radius: $\sqrt{(2)^2+(-3)^2-5}=2\sqrt{2}$

(ii) Centre: $(2\,,-1)$

Radius: $\sqrt{72}=6\sqrt{2}$

Distance between centres: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\sqrt{\left(2-(-2)\right)^2+(-1-3)^2}=\sqrt{32}=4\sqrt{2}$

The difference the two radii as the circles touch internally: $6\sqrt{2}-2\sqrt{2}=4\sqrt{2}$

$4\sqrt{2}=4\sqrt{2}$

(b) Distance between: $(7\,,10)$ and $(9\,,k)$

Distance between two points: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\sqrt{(9-7)^2+(k-10)^2}$

Same for $(12\,,8)$ and $(9\,,k)$

$\sqrt{(9-12)^2+(k-8)^2}$

$\sqrt{(9-12)^2+(k-8)^2}=\sqrt{(9-7)^2+(k-10)^2}$

$(3)^2+(k-8)^2=(2)^2+(k-10)^2$

$9+k^2-16k+64=4+k^2-20k+100$

$4k=31$

$k=\frac{31}{4}$

Centre: $\left(9\,,\frac{31}{4}\right)$

Distance between centre and point: $\sqrt{(9-7)^2+\left(\frac{31}{4}-10\right)^2}=\frac{\sqrt{145}}{4}$

$(x-9)^2+\left(y-\frac{31}{4}\right)^2=\frac{145}{16}$

(a) $(1\,,13) \rightarrow +5$ and $-2 \rightarrow (6\,,11)$

$(6\,,11) \rightarrow\ +3(5)$ and $3(-2) \rightarrow B(21\,,5)$

(b) Perpendicular distance formula: $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$\frac{|\frac{4}{3}(5)+(-1)(-2)-11|}{\sqrt{\left(\frac{4}{3}\right)^2+(-1)^2}} = \frac{|-7|}{5} = \frac{7}{5} = 1.4$ units

(c) (i) $\binom{16}{2} = 120$

(ii) $\binom{4}{2} \times 4 = 24$

$\frac{24}{120} = \frac{1}{5}$

(a) (i) $z = \frac{50-48.2}{10.6} = 0.1698$

$P(z < 0.1698) = 0.5675 = 56.75\%$

(ii) $10\% \rightarrow 100 - 10 = 90\%$

Probability: 0.9000 corresponding z-value: 1.28

$1.28 = \frac{A-48.2}{10.6}$

$13.568 = A - 48.2$

$A = 62$ (nearest whole number)

(b) (i) 2 successes: $\binom{6}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^4 = \frac{768}{3125} = 0.24576$

(ii) $\binom{n}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^n = 0.0047$

$\left(\frac{4}{5}\right)^n = 0.0047$

$n = log_{\frac{4}{5}}(0.0047) = 24$ (nearest whole number)

(c) Booked: $0.45\left(\frac{1}{3}\right) + 0.55\left(\frac{2}{5}\right) = \frac{37}{100}$

Booked on new system: $0.55\left(\frac{2}{5}\right) = \frac{11}{50}$

Probability: $\frac{\frac{11}{50}}{\frac{37}{100}} = \frac{22}{37} = 59\%$

(d) Null Hypothesis: $p = 0 \cdot 75$
Alternate hypothesis: $P \neq 0.75$

Confidence interval: $0.765 \pm \frac{1}{\sqrt{1000}} = 0.7334 < p < 0.7966$

Conclusion: fail to reject H_0

There is not enough evidence to conclude that this percentage has changed.

(a) One of the lengths: 15

The other: $2\pi(5) = 31.4$

Answer: 15×31.4

(b) $r^2 = 6^2 + 11^2$

$r = \sqrt{36 + 121} = \sqrt{157}$

Volume of the sphere: $\frac{4}{3}\pi r^3 \rightarrow \frac{4}{3}\pi r^3$

$= 8{,}240.2 cm^3$

(c) (i) Opposite angles in a cyclic quadrilateral sum to $180°$

(ii) $Angle\ CEB = Angle\ DEB$ as both $90°$
$Angle\ ECB = 90 - Angle\ EDB$
$Angle\ DBE = 90 - Angle\ EDB$
Therefore: $Angle\ ECB = Angle\ DBE$
So $angle\ CBE = angle\ EDB$ as the angles in a triangle sum to $180°$

(iii) Similar triangles: $\frac{a_1}{b_1} = \frac{a_2}{b_2}$
$\frac{r}{20-h} = \frac{h}{r}$

$r^2 = h(20 - h)$

$r^2 = 20h - h^2$

(iv) Volume of the cone: $\frac{1}{3}\pi r^2 h$

Sub in for r^2

$\frac{1}{3}\pi(20h - h^2)(h)$

$\frac{20h^2\pi}{3} - \frac{h^3\pi}{3}$

First derivative: $\frac{40h\pi}{3} - h^2\pi = 0$
$40h - 3h^2 = 0$
$h(40 - 3h) = 0$
$h = 0$ or $3h = 40$

$h = \frac{40}{3}$ cm (as with $h = 0$, the cone would not exist)

(a) (i) $(x-1)^2 + (y-17)^2 = 144$

(ii) (sub in point:$(a\,,8)$)

$(a-1)^2 + (8-17)^2 = 144$

$(a-1)^2 + 81 = 144$

$(a-1)^2 = 63$

$a - 1 = \sqrt{63}$

$a = \sqrt{63} + 1$

(iii) Find distance from point to centre and then subtract radius

Distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(10-1)^2 + (6-17)^2} = \sqrt{202}$

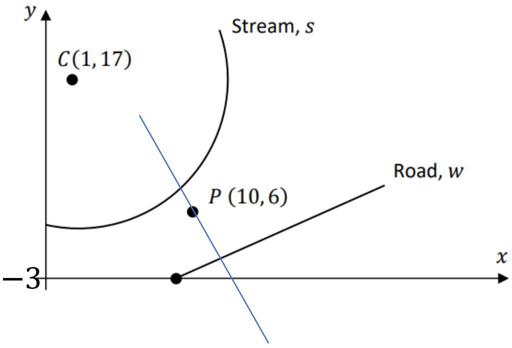
$\sqrt{202} - radius$

$\sqrt{202} - 12 = 2.2126$ $\quad\quad \times 100$

$221m$

(b) $1 + radius \rightarrow 1 + 12 = 13$

Equation of the line: $x = 13$



(c) Find slope of W: $y = mx + c \rightarrow y = \frac{x}{3} - 3$

Find perpendicular slope and make an

equation of the line through point $(10, 6)$

Slope: $m_1 \times m_2 = -1 \rightarrow \frac{1}{3}(m_2) = -1 \rightarrow m_2 = -3$

$y - y_1 = m(x - x_1)$

$y - 6 = -3x + 30$

Simultaenous equations to find point closest to $(10, 6)$

$3x + y = 36$ multiply by 3

$x - 3y = 9$

$9x + 3y = 108$

$x - 3y = 9$

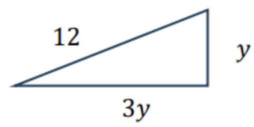
$10x = 117$

$x = 11.7$

$(11.7) - 3y = 9$

$y = 0.9$

$(11.7, 0.9)$



(d) Slope is the ratio of the opposite and alternate sides:

$y^2 + (3y)^2 = 12^2$

$y^2 + 9y^2 = 144$

$10y^2 = 144$

$y^2 = 14.4 \rightarrow y = \sqrt{14.4} = 3.8$

$x = 3y + 9$ (starting point $(9, 0)$, therefore add 9)

$x = 3(\sqrt{14.4}) + 9 = 20.4$

$(20.4, 3.8)$

(a) (i) $|OB| = |ON| = 100 + 20 = 120$ QED

(ii) $Cosx = \frac{adjacent}{hypotenuse} \rightarrow \frac{90}{120}$

$x = \cos^{-1}\frac{90}{120} = 41.4°$

(iii) Find $|AO|$

$Sinx = \frac{opposite}{hypotenuse}$

$Sin(41.4) = \frac{20}{x}$

$x = \frac{20}{\sin(41.4)} = 30.24$

Area of sector: $\frac{\theta}{360}\pi r^2$

Area of BOB' – Area of AOA'

$\frac{97.2}{360}\pi(120)^2 - \frac{97.2}{360}\pi(30.24)^2 = 11{,}439cm^2$

(b) Cosine rule: $a^2 = b^2 + c^2 - 2bcCosA$

$|OE| = |OE'| = x$

$180^2 = x^2 + x^2 - 2(x)(x)Cos(105)$

$180^2 = 2x^2 - 2x^2Cos(105)$

$32{,}400 = x^2(2 - 2Cos105)$

$x^2 = \frac{32{,}400}{2-2Cos105}$

$x = \sqrt{\frac{32{,}400}{2-2Cos105}}$

$x = \frac{180}{\sqrt{2-2Cos105}} = 113.4cm$

(c) (i) $3^5 = 243$

(ii) $1 \times 3^3 \times 2 = 54$

(iii) $3 \times 2 \times 2 \times 2 \times 2 = 48$