(a) $b^2 - 4ac = 0$ (formula for nature of roots, when one real root)

$ax^2 + bx + c = 0$

$a = 3$, $b = -m$, $c = 3$

$(-m)^2 - 4(3)(3) = 0$

$m^2 - 36 = 0$

$m^2 = 36$

$m = \pm 6$

(b) $4x^2 + 12x + 9 + 7 = 0$ (multiply out equation)

$4x^2 + 12x + 16 = 0$

$ax^2 + bx + c = 0$

$a = 4$, $b = 12$, $c = 16$

$b^2 - 4ac$ (formula for nature of roots)

$12^2 - 4(4)(16)$

$144 - 256 = -112$

$b^2 - 4ac < 0$ therefore no real roots

(c) (i) $3(-1)^2 + 2(-1) + 5 = 0$

$3 - 2 + 5 \neq 0$

(ii) $(x + 1)(ax + b) + c$

$ax^2 + ax + bx + b + c$

$3x^2 + 2x + 5$ (compare the coefficients of each equation)

$ax^2 = 3x^2$, $2x = ax + bx$, $b + c = 5$

$a = 3$, $2 = a + b$, $b + c = 5$

$2 = 3 + b \rightarrow \; b = -1$

$(-1) + c = 5 \rightarrow \; c = 6$

(a) $\frac{2}{3}x^3 + \frac{5}{2}x^2 + 6x + c$

(b) (i) $\int_0^2 ax^2 + bx + c \;\; dx$

$$\left[\frac{a}{3}x^3 + \frac{b}{2}x^2 + cx\right]_0^2$$

$$\frac{a}{3}(2)^3 + \frac{b}{2}(2)^2 + c(2) - \left(\frac{a}{3}(0)^3 + \frac{b}{2}(0)^2 + c(0)\right) = 538$$

$\frac{8a}{3} + 2b + 2c = 538$ (multiply by 3 and divide by 2 to get simplest form)

$$\frac{8a+6b+6c=1614}{2}$$

$4a + 3b + 3c = 807$ QED

(ii) 1. $4a + 3b + 3c = 807$ (multiply by -1)

2. $28a + 9b + 3c = 879$

3. $76a + 15b + 3c = 663$

2. $28a + 9b + 3c = 879$

$-1.$ $-4a - 3b - 3c = -807$

4. $24a + 6b = 72$

3. $76a + 15b + 3c = 663$

$-1.$ $-4a - 3b - 3c = -807$

5. $72a + 12b = -144$

4. $24a + 6b = 72$ (multiply by -2)

4. (-2) $-48a - 12b = -144$

5. $72a + 12b = -144$

$24a = -288$

$a = -12$

4. $24(-12) + 6b = 72$ (subbed in for 'a')

$-288 + 6b = 72 \rightarrow 6b = 360 \rightarrow b = 60$

1. $4(-12) + 3(60) + 3c = 807$ (subbed in for ‘a’ and ‘b’)

   $-48 + 180 + 3c = 807$

   $3c = 675$

   $c = 225$

   $a = -12$, $b = 60$, $c = 225$

(a) (i) $z = 6 + 2i$

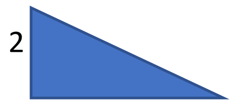
$z - iz \rightarrow (6+2i) - i(6+2i)$

$6 + 2i - 6i - 2i^2$

$6 + 2i - 6i + 2$

$8 - 4i$

(ii) $|z|$ = length of the radius of z in an argand diagram



$a^2 + b^2 = c^2$

$6^2 + 2^2 = c^2$

$c = |z| = \sqrt{40}$

$|iz| \rightarrow iz = 2 - 6i$

$2^2 + (-6)^2 = c^2$

$|zi| = \sqrt{40}$

$|z - iz|$

$4^2 + 8^2 = c^2$

$|z - iz| = \sqrt{80}$

$|z|^2 + |iz|^2 = |z - iz|^2$

$(\sqrt{40})^2 + (\sqrt{40})^2 = (\sqrt{80})^2$

$40 + 40 = 80$ QED

(iii) Find distance from z to iz (treat $6 + 2i$ as a point, $x = 6$, $y = 2$)

Distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Points: $6 + 2i$, $2 - 6i$

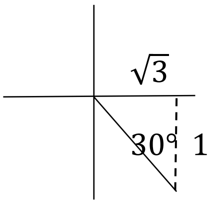
$\sqrt{(6-2)^2 + (2-(-6))^2} = \sqrt{80}$

Diameter $= \sqrt{80}$, radius $= \frac{\sqrt{80}}{2} = \sqrt{20}$

Area: $\pi r^2 \rightarrow \pi\left(\sqrt{20}\right)^2$

20π

(b)



$a^2 + b^2 = c^2$

$(\sqrt{3})^2 + (1)^2 = r^2$

$r = \sqrt{4} = 2$

$\theta = 360 - 30° = 330°$

De Moivre's theorem: $r(cos\theta + isin\theta)$

$2(cos330 + isin330)$

$(2(cos330 + isin330))^9$

$2^9(\cos(330(9)) + isin(330(9)))$

$512(cos2970 + isin2970)$

$= 0 + 512i$

$a = 0\ ,\ b = 512$

(a) $u_{n+1} = \sqrt{\frac{u_n}{u_{n-1}}}$

$u_{2+1} = \sqrt{\frac{u_2}{u_{2-1}}} = \sqrt{\frac{64}{2}} = \sqrt{32} = \sqrt{2^5} = 2^{\frac{5}{2}}$

(b) (i) Arithmetic sequence: 3rd term − 2nd term = 2nd term − 1st term

$\frac{5}{e^k} = \frac{5}{y}$, 13 , $5e^k = 5y$

$5y - 13 = 13 - \frac{5}{y}$

$5y - 26 + \frac{5}{y} = 0$

$5y^2 - 26y + 5 = 0$ QED

(ii) $5y^2 - 26y + 5 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad a = 5$, $b = -26$, $c = 5$

$\frac{-(-26) \pm \sqrt{(-26)^2 - 4(5)(5)}}{2(5)} = \frac{26 \pm 24}{10} = y = 5 \ or \ 0.2$

Put $y = e^k$

$5 = e^k \quad \rightarrow \quad ln5 = k$

$0.2 = e^k \quad \rightarrow \quad \ln 0.2 = k = ln\frac{1}{5} = ln5^{-1} = -ln5$

$k = ln5 \ or - ln5$

(a) $g(x) = x^2 - x^{-1}$

$g'(x) = 2x + x^{-2}$

$g'(x) = 2x + \frac{1}{x^2}$

(b) (i)

$$\begin{array}{r|l} & 2x^2 - 23x + 63 \\ \hline x + 1 & 2x^3 - 21x^2 + 40x + 63 \\ & -(2x^3 + 2x^2) \\ & \quad -23x^2 + 40x \\ & \quad -(-23x^2 - 23x) \\ & \qquad 63x + 63 \\ & \qquad -(63x + 63) \\ & \qquad\qquad 0 \end{array}$$

$2x^2 - 23x + 63 = 0$

$(2x - 9)(x - 7) = 0$

$2x = 9 \quad or \quad x = 7$

$x = \frac{9}{2} \quad or \quad x = 7 \quad or \quad x = -1$

(ii) $f'(x) = 6x^2 - 42x + 40$

$6x^2 - 42x + 40 = 0$ (Put $= 0$, to find points at which the function's outputs become positive)

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a = 6 \ , \ b = -42 \ , \ c = 40$

$\frac{-(-42) \pm \sqrt{(-42)^2 - 4(6)(40)}}{2(6)} = \frac{42 \pm \sqrt{1764 - 960}}{12} = \frac{42 \pm 2\sqrt{201}}{12} = 5.862 \ or \ 1.137$

$1.14 < x < 5.86$

(a) $f(x+h) = 2(x+h)^2 + 4(x+h)$

$f(x) = 2x^2 + 4x$

$f(x+h) - f(x) = 2(x+h)^2 + 4(x+h) - (2x^2 + 4x)$

$f(x+h) - f(x) = \quad 2x^2 + 4hx + 2h^2 + 4x + 4h - 2x^2 - 4x$

$= 4hx + 2h^2 + 4h$

$\frac{f(x+h)-f(x)}{h} = \frac{4hx+2h^2+4h}{h}$

$= 4x + 2h + 4 \qquad h\ limit\ 0$

$= 4x + 2(0) + 4$

$f'(x) = 4x + 4$

(b) Length: $4x$

Width: x

Area of the rectangle: $(4x)(x) = 4x^2$

$4x^2 = 225$

$x = \sqrt{\frac{225}{4}} = 7.5$

Rate of change of the area: $8x$

$8(7.5) = 60\text{cm}^2/\text{cm}$ (cm^2 of area increased per cm of width increased)

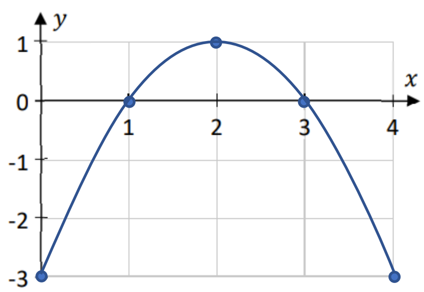
(c) Point given by question: $(0, -3)$

Same slope at start and at end for $p(x)$, therefore: $(4, -3)$

Graph of $p'(x)$ cuts at $(1, 0)$ and $(3, 0)$ as they are turning points

Point of inflection is maximum point for $p'(x)$ therefore it is: $(2, 1)$

Graph of $y = p'(x)$



(a) $h(4) = 2(4)^3 - 28.5(4)^2 + 105(4) + 70 = 162$ BPM

(b) $h'(x) = 6x^2 - 57x + 105$

(c) $h'(2) = 6(2)^2 - 57(2) + 105 = 15$

Explanation: the rate that Hannah's heart rate is increasing after 2 minutes

(d) $h'(x) = 6x^2 - 57x + 105 = 0$

$2x^2 - 19x + 35 = 0$

$(2x - 5)(x - 7) = 0$

$2x = 5 \; or \; x = 7$

$x = 2.5 \; or \; 7$ (Sub back into $h(x)$ to find values)

$h(2.5) = 2(2.5)^3 - 28.5(2.5)^2 + 105(2.5) + 70 = 185.625$ (greatest value)

We know the least value is when $x = 0$ from looking at the graph

$h(0) = 2(0)^3 - 28.5(0)^2 + 105(0) + 70 = 70$ (least value)

(e) $h'(x) = 6x^2 - 57x + 105$

$h''(x) = 12x - 57 = 0$ (find the point of inflection)

$12x = 57$

$x = \frac{57}{12} = 4.75 \; or$ 4 minutes and 45 seconds

(f) (i) $b'(x) = h'(x)$

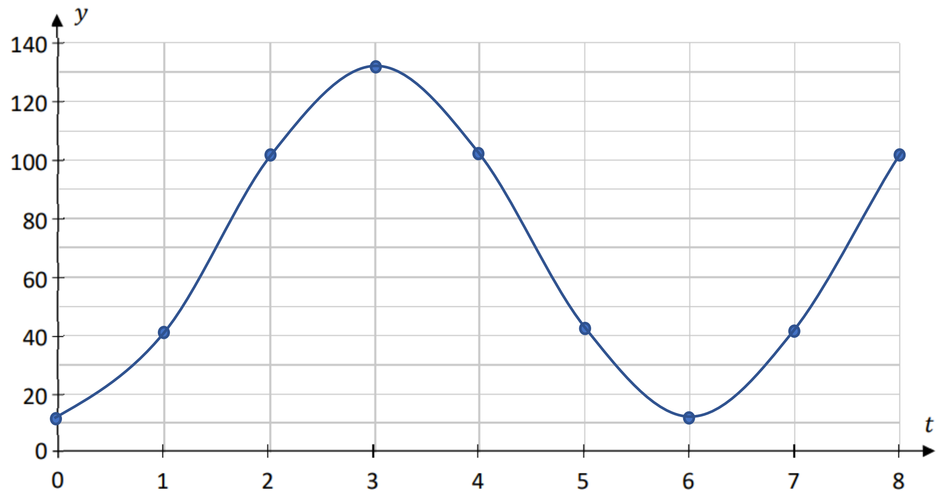
(ii) $0.9h'(x)$

(g) $2(0.8x)^3 - 28.5(0.8x)^2 + 105(0.8x) + 70$

$m(x) = 1.024x^3 - 18.24x + 84x + 70$

(a) $12\ ,\ 42\ ,\ 102\ ,\ 132\ ,\ 102\ ,\ 42\ ,\ 12\ ,\ 42\ ,\ 102$

(b)



(c) Period: 6

Range: $[12\ , 32]$

(d) Time above 42 in a period: 4

$50 \div 6\ (minutes) = 8$ periods and 2 extra minutes

$8 \times 4 = 32 + 2$ (extra minutes) $= 34$ minutes

(e) $72 - 60\cos\left(\frac{\pi}{3}t\right) = 110$

$-60cos\left(\frac{\pi}{3}t\right) = 38$

$cos\left(\frac{\pi}{3}t\right) = -\frac{38}{60}$

$\frac{\pi}{3}t = cos^{-1}\left(\frac{38}{60}\right)$

$\frac{\pi}{3}t = \pi + 0.8849$ or $\frac{\pi}{3}t = \pi - 0.8849$ (cos is negative in quadrant 2 and 3)

$t = \frac{3(\pi+0.8849)}{\pi} = 3.85$ mins or $t = \frac{3(\pi-0.8849)}{\pi} = 2.15$

2nd value: 3.85 mins

(f) $\frac{1}{8}\int_0^8 \left(72 - 60\cos\left(\frac{\pi}{3}t\right)\right)$

$\frac{1}{8}\left\{\begin{matrix}8\\0\end{matrix}\right. 72t - 60\left(\frac{3}{\pi}\right)\sin\left(\frac{\pi}{3}t\right)$

$\frac{1}{8}\left(\left(72(8) - \frac{180}{\pi}\sin\left(\frac{\pi}{3}(8)\right)\right) - \left(72(0) - \frac{180}{\pi}\sin\left(\frac{\pi}{3}(0)\right)\right)\right)$

$72 - 6.2 = 65.8$ m

(a) $15(0.6)^{2.5} = 4.18$ mg

(b) $15(0.6)^t = 1$

$0.6^t = \frac{1}{15}$

$t = log_{0.6}\left(\frac{1}{15}\right) = 5.3$ days

(c) There are 4, 15 mg injections, but they each are decaying at a rate of 0.6 per day

Therefore for each 15 mg injection, the 4 of them have decayed by nothing, 0.6, 0.6^2, 0.6^3

(d) $S_n = \frac{a(1-r^n)}{1-r}$ $\quad a = 15\,, r = 0.6\,, n = 10$

$S_{10} = \frac{15(1-0.6^{10})}{1-0.6} = 37.27$ mg

(e) $S_\infty = \frac{a}{1-r} = \frac{15}{1-0.6} = 37.5$ mg

(f) (i) $d + d(0.85) + d(0.85)^2 + \cdots + d(0.85)^{n-1}$

$S_n = \frac{a(1-r^n)}{1-r}$ $\quad a = d\,, r = 0.85$

$S_n = \frac{d(1-0.85^n)}{1-0.85} = \frac{d(1-0.85^n)}{0.15}$

$\frac{20}{20} \times \frac{d(1-0.85^n)}{0.15} = \frac{20d(1-0.85^n)}{3}$

(ii) $\frac{20d(1-0.85^7)}{3} = 50$

$d(1 - 0.85^7) = \frac{150}{20}$

$d = \frac{150}{20(1-0.85^7)} = 11.03$ mg

$d = 11$ mg

(a) $P(3) = 0.82 - 0.12\ln(3+1) = 0.65$

(b) $0.82 - 0.12\ln(t+1) = 0.55$

$0.12\ln(t+1) = 0.27$

$\ln(t+1) = \frac{0.27}{0.12}$

$t + 1 = e^{\frac{0.27}{0.12}}$

$t = e^{2.25} - 1 = 8.49$ hours

(c) (i) $P'(t) = -\frac{0.12}{t+1}$

$P'(1) = -\frac{0.12}{1+1} = -0.06$

(ii) The proportion decreases as 't' goes up

(d) $P'(t) = -0.12(t+1)^{-1}$ (second derivative used to find point of inflection)

$P''(t) = 0.12(t+1)^{-2} = 0$ (put $= 0$ to find point of inflection)

$0.12 \neq 0$ therefore no point of inflection

(e) (i) $log_{10}A = log_{10}(B(t+1)^c)$

$log_{10}A = log_{10}B + log_{10}(t+1)^c$

$(c)log_{10}(t+1) = log_{10}A - log_{10}B$

$c = \frac{log_{10}A - log_{10}B}{log_{10}(t+1)}$

(ii) $c = \frac{log_{10}(80) - log_{10}(47)}{log_{10}(24+1)} = 0.165$