(a) (i)

	Age (years) ≤ 23	Age (years) ≥ 24	Total
Under.	12 785	2922	15 707
Post.	1353	**5654**	**7007**
Total	**14 138**	8576	22 714

(ii) $P(O) = \frac{8{,}576}{22{,}714} = 0.377$

$P(U) = \frac{15{,}707}{22{,}714} = 0.6915$

$P(O) \times P(U) = P(O \cap U)$ if events are independent

$0.377 \times 0.6915 = 0.261$

$P(O \cap U) = \frac{2{,}922}{22{,}714} = 0.129$

$0.261 \neq 0.129$

Therefore events are not independent

(b) $\frac{1}{7} \times \frac{1}{7} \times \frac{1}{7} \times 7 = \frac{1}{49}$

(c) $\frac{g}{b+g} = \frac{3}{5} \quad \to \quad 5g = 3b + 3g \quad \to \quad 3b = 2g \quad \to \quad b = \frac{2}{3}g$

$\frac{g+4}{b+g+8} = \frac{4}{7}$

$\frac{g+4}{\frac{2}{3}g+g+8} = \frac{4}{7}$

$\frac{g+4}{\frac{5}{3}g+8} = \frac{4}{7}$

$7(g+4) = 4\left(\frac{5}{3}g + 8\right)$

$7g + 28 = \frac{20}{3}g + 32$

$7g - \frac{20}{3}g = 32 - 28$

$\frac{1}{3}g = 4$

$g = 12$

$\frac{12}{b+12} = \frac{3}{5}$

$60 = 3b + 36 \quad \to \quad 3b = 24 \quad \to \quad b = 8$

(a) Internal divisor formula: $\left(\frac{b(x_1)+a(x_2)}{a+b}, \frac{b(y_1)+a(y_2)}{a+b}\right)$

$\left(\frac{1(8)+4(-1)}{4+1}, \frac{1(-4)+4(3)}{4+1}\right)$

$\left(\frac{4}{5}, \frac{8}{5}\right)$

(b) $y - y_1 = m(x - x_1)$

$y - r = m(x - q)$

$y = mx - mq + r$

$y = m(0) - mq + r$ (the graph intercepts the y-axis, when $x = 0$)

$(0, -mq + r)$

(c) $tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

$tan30 = \frac{-2 - m_2}{1 + (-2)m_2}$

$\frac{1}{\sqrt{3}} = \frac{-2 - m_2}{1 - 2m_2}$

$1 - 2m_2 = -2\sqrt{3} - \sqrt{3}m_2$

$\sqrt{3}m_2 - 2m_2 = -2\sqrt{3} - 1$

$m_2(\sqrt{3} - 2) = -2\sqrt{3} - 1$

$m_2 = \frac{-2\sqrt{3} - 1}{\sqrt{3} - 2}$

$m_2 = \frac{2\sqrt{3} + 1}{2 - \sqrt{3}}$

$m_2 = \frac{2\sqrt{3} + 1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$ (Multiply by the conjugate, to remove fractions from irrational numbers)

$m_2 = \frac{2(2\sqrt{3}+1) + \sqrt{3}(2\sqrt{3}+1)}{2(2-\sqrt{3}) + \sqrt{3}(2-\sqrt{3})}$

$m_2 = \frac{4\sqrt{3} + 2 + 2(\sqrt{3})^2 + \sqrt{3}}{4 - 2\sqrt{3} + 2\sqrt{3} - (\sqrt{3})^2}$

$m_2 = \frac{5\sqrt{3} + 6 + 2}{4 - 3} = 5\sqrt{3} + 8$

(a) Centre: $\frac{-(-2)}{2}, \frac{-(8)}{2} \rightarrow (1, -4)\ (-g, -f)$

Radius: $\sqrt{g^2 + f^2 + c}$

$\sqrt{(-1)^2 + 4^2 - k} = 5\sqrt{3}$

$\sqrt{1 + 16 - k} = 5\sqrt{3}$

$\left(\sqrt{1 + 16 - k}\right)^2 = \left(5\sqrt{3}\right)^2$

$17 - k = 25(3)$

$-k = 75 - 17$

$k = -58$

(b) $(5, -2)$

Slope of the line formula: $\frac{y_2 - y_1}{x_2 - x_1} \rightarrow \frac{-4-(-2)}{9-5} = -\frac{1}{2}$

Tangent is perpendicular to this line

Perpendicular slopes: $m_1 \times m_2 = -1$

$-\frac{1}{2} \times m_2 = -1$

$m_2 = 2$

(c) Centre of circle: $(r, -r)$

Sub into equation: $(x - h)^2 + (y - k)^2 = r^2$ ($r =$ radius)

$(1 - r)^2 + (-8 - (-r))^2 = r^2$

$1 - 2r + r^2 + 64 - 16r + r^2 = r^2$

$r^2 - 18r + 65 = 0$

$(r - 5)(r - 13) = 0$

$r = 5 \ \ or \ \ r = 13$

Equation of small circle: $(x - 5)^2 + (y + 5) = 25$

Equation of big circle: $(x - 13)^2 + (y - 13)^2 = 169$

(a) (i) $tan(A+(-B)) = \frac{\tan(A)+\tan(-B)}{1-tan(A)tan(-B)}$ $\tan(-B) \rightarrow -\tan(B)$

$\frac{tanA-tanB}{1+tanAtanB}$

(ii) $tan(60-45) = \frac{tan60-tan45}{1+tan60tan45}$

$\frac{\sqrt{3}-1}{1+\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

(b) Angle ABC $= \frac{180-45}{2} = 67.5°$ (isosceles triangle)

Sine rule: $\frac{a}{sinA} = \frac{b}{sinB}$

$\frac{x}{sin67.5} = \frac{10\sqrt{2-\sqrt{2}}}{sin45}$

$x = \frac{(10\sqrt{2-\sqrt{2}})(sin67.5)}{sin45} = 10$

(a) (i) $\hat{p} = \frac{135}{400} = 0.3375$

(ii) $\hat{p} \pm \frac{1}{\sqrt{n}}$

$0.3375 \pm \frac{1}{\sqrt{400}}$

0.3375 ± 0.05

$0.2875 < p < 0.3875$

(iii) $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$0.3375 \pm 1.96\sqrt{\frac{0.3375(1-0.3375)}{400}}$

0.3375 ± 0.04633

$0.2912 < p < 0.3838$

(b) *Null hypothesis*: the average amount has not changed

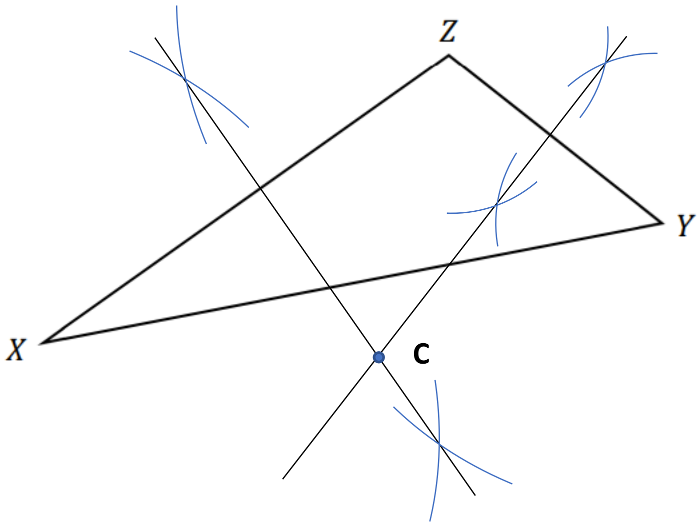
Alternate hypothesis: the average amount has changed

Conclusion: the average amount has changed

Caclulation and reasoning: $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \rightarrow \frac{22.16-20.79}{\frac{8.12}{\sqrt{500}}} = 3.7726$

z-values outside the range of: $-1.96 < z < 1.96$

(a)



(b) Angle ADB $= 90°$ (angle in a semi-circle)

Angle ABD $= 45°$ (isosceles triangle) $(\frac{180-90}{2})$

Angle ACD $= 45°$ (angles on the same arc)

Angle ADC $= 180° - 40° - 45° = 95°$

(c) Assume that O is inside the triangle PQR.

Then Angle PQR $< \ 180°$, where Angle POQ is the angle in the triangle POQ.

But Angle PRQ is half this angle, so Angle PRQ $< \ 90°$, a contradiction.

(a) Radius $= \frac{10}{2} = 5$ cm

Volume of a cylinder: $\pi r^2 h$

$\pi(5)^2 h = 450\pi$

$h = \frac{450\pi}{25\pi} = 18$ cm

(b) Volume of a cone: $\frac{1}{3}\pi r^2 h$

Small: $\frac{1}{3}\pi r^2 h = 12\pi \quad \rightarrow \quad r^2 h = 36$

Large: $\frac{1}{3}\pi (kr)^2 (2h) = 150\pi \quad \rightarrow \quad r^2 h = 225k^2$

$36 = 225k^2$

$k^2 = \frac{225}{36}$

$k = \sqrt{\frac{225}{36}} = 2.5$

(c) Length of arc formula: $\frac{\theta}{360}(2\pi r)$

$\frac{216}{360}\left(2\pi(8)\right) = 9.6\pi$ cm

Circumference formula: $2\pi r$

$2\pi r = 9.6\pi$

$r = 4.8$ cm

(d) (i) Volume of a sphere: $\frac{4}{3}\pi r^3$

$\frac{4}{3}\pi(2.7)^3 = 82.448$ cm^3

(ii) Radius of flat circle: $5.4 = \pi r^2$

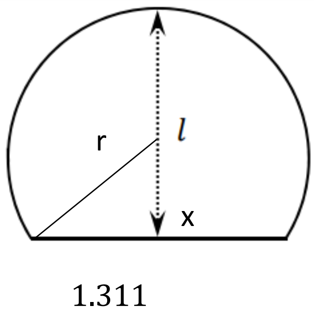
Radius$= 1.311$

$a^2 + b^2 = c^2$

$1.311^2 + x^2 = 2.7^2$

$x = \sqrt{2.7^2 - 1.311^2} = 2.36$

$2.7 + 2.36 = 5.06 = 5.1$ cm



(e) $|\text{EB}| = 15$

$|\text{EC}| = \sqrt{15^2 + 30^2} = 15\sqrt{5}$ (Pythagoras)

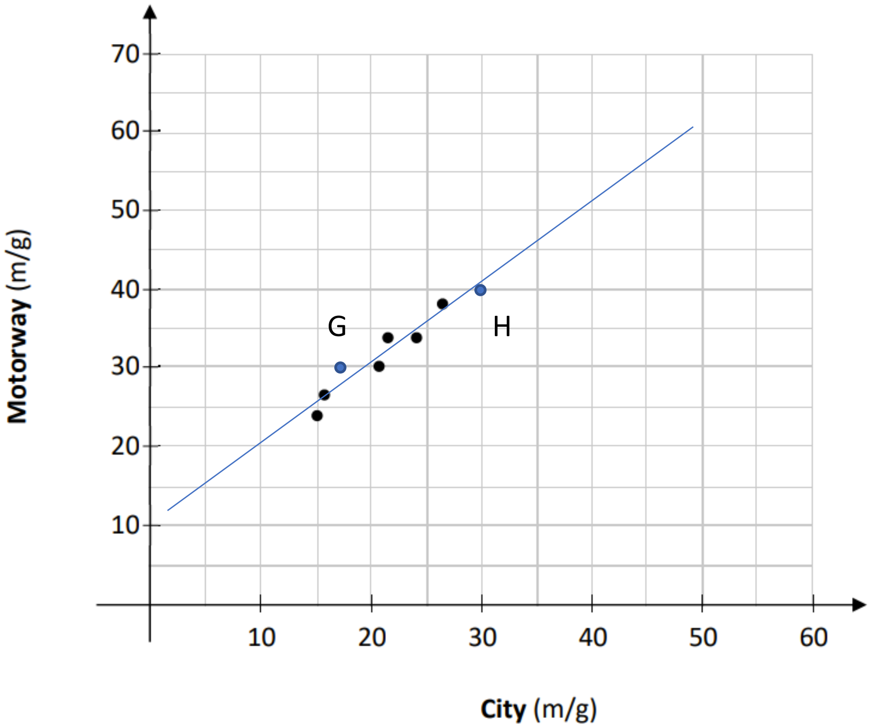
Similar triangles: EOB and ECB as shared angles

$\frac{EO}{EB} = \frac{EB}{EC}$

$EO = 15\left(\frac{15}{15\sqrt{5}}\right) = 3\sqrt{5}$ cm

(a) (i)

(ii)



(iii)

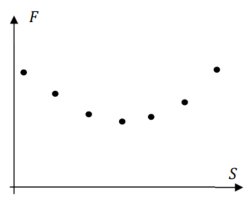
Car	City (m/g)	Motorway (m/g)
K	20	30
L	48	60

(iv) K

L is beyond all of the given data points by a large amount

(v) $r = 0.966$

(b) The line of best fit is close to horizontal



(c) Mean: $\frac{534+S+M}{13} = 52 \quad \rightarrow \quad S + M = 676 - 534 = 142$

Median: 54, 7th number when ordered, therefore S or M has to be after it

Least value after 54 is 55 $\quad S = 55$ Least

Or greatest value is when M is at it's smallest and that can only be 55, therefore $S + 55 = 142$

$S = 77$ Greatest

(d) (i) $20{,}000 \times 0.095 = €1{,}900$ if not replaced

$20{,}000 \times 0.005 + 1{,}450 = €1{,}550$ if replaced

It is worth replacing the gasket now

(ii) $(1 - 0.095) \times (1 - 0.041) \times (1 - 0.073) = 0.8045$ prob of 0 events happening

$1 - 0.805 = 0.195$

(a) Field 1

$\frac{1}{2}abSinC$

$\frac{1}{2}(35)(30)Sin50 = 402$ m^2

Field 2

$\frac{1}{2}abSinC$ $\quad b = |AB| + |BD|$

$\frac{1}{2}(35)(30+10)Sin50 - 402$ (402 is the area of field 1)

$= 134$ m^2

(b) Cosine rule: $a^2 = b^2 + c^2 - 2bcCosA$

$x^2 = (30)^2 + (35)^2 - 2(30)(35)Cos50 = 775.146$

$x = \sqrt{775.146} = 28$ m

Perimeter: $28 + 30 + 35 = 93$ m

(c) (i) $cosX = \frac{adjacent}{hypotenuse} \rightarrow \quad Cos45 = \frac{10}{X}$

$X = \frac{10}{Cos45} = 14.142$ km

$\frac{14.142}{343} \times 1{,}000 = 41.23 = 41$ seconds

(ii) $41 \times 255 = 10{,}455$ m

$10.455 - 10 = 0.455$ km

$tan\theta = \frac{opposite}{adjacent} \rightarrow \quad tan\theta = \frac{0.455}{10}$

$\theta = tan^{-1}\frac{0.455}{10} = 2.6°$

(d) (i) For example: The time taken for the sound to go from P3 to O [LHS] is the same as the time taken for the plane to go from P3 to P4 [RHS]

(ii) $\frac{2d}{0.255} \times 0.343 = \sqrt{100 + d^2}$

$\left(\frac{686d}{255}\right)^2 = 100 + d^2$

$7.237d^2 = 100 + d^2$

$6.237d^2 = 100$

$d^2 = \frac{100}{6.237}$

$d = \sqrt{\frac{100}{6.237}} = 4.004 = 4$

(a) (i) $z = \frac{x-\mu}{\sigma} \quad \rightarrow \quad \frac{240-225}{12} = 1.25$

$P(z < 1.25) = 0.8944$

$P(z > 1.25) = 1 - 0.8944 = 0.1056 = 10.56\%$

(ii) $1 - 0.2 = 0.8$

$P(z < x) = 0.8000$ (find corresponding z-value)

$z = 0.84$

$\frac{x-225}{12} = -0.84$

$x - 225 = -0.84(12)$

$x = 225 - 0.84(12) = 214.8 = 215$ seconds

(b) $\binom{3}{0}(0.05)^0(1-0.05)^3 = 0.8574$ (probability of no false starts in her first 3 races)

$0.8574 \times 0.05 = 0.0429$ (false start in next race)

(c) $P(0), P(1), P(2)$

$\binom{20}{0}(0.1)^0(1-0.1)^{20} + \binom{20}{1}(0.1)^1(1-0.1)^{19} + \binom{20}{2}(0.1)^2(1-0.1)^{18} = 0.6769$

(d) $1 + 100, 2 + 99, 3 + 98, \ldots, 50 + 51$ (50 total pairs that fit this combination)

$\binom{300}{2} = 44{,}850$ pairs

$\frac{50}{44{,}850} = \frac{1}{897}$

(e) $\frac{5265}{6000} = 0.8775$ (proportion of people who ran the marathon faster than her)

Corresponding z-value: 1.16

$\frac{x-254}{38} = 1.16$

$x = 254 + 1.16(38) = 298.08$ (finishing time for both marathons)

$\frac{298.08-247}{29} = 1.76$ (z-value)

Corresponding P-value: 0.9608 (proportion of people who ran the marathon faster than her)

$0.9608 \times 2{,}000 = 1{,}921.6$

$1{,}922^{\text{nd}}$