**(a)** $|5 + 3m|^2 = 11^2$

$25 + 30m + 9m^2 = 121$

$9m^2 + 30m - 96 = 0$

$3m^2 + 10m - 32 = 0$

$(3m + 16)(m - 2) = 0$

$3m + 16 = 0$ , $m - 2 = 0$

$m = -\frac{16}{3}$ , $m = 2$

**(b)** $\frac{1}{h} = \frac{k}{k+j}$

$\frac{1(k+j)}{h} = k$ (multiply both sides by $(k + j)$)

$k + j = kh$

$k - kh = -j$

$k(1 - h) = -j$

$k = -\frac{j}{1-h}$

**(c)** $(x^2 - px + 1)(x + k) = x^3 - 2x - 3r$

$x(x^2 - px + 1) + k(x^2 - px + 1)$

$x^3 - px^2 + x + kx^2 - kpx + k$

$x^3 - px^2 + kx^2 + x - kpx + k = x^3 - 2x - 3r$ (match 'x' terms)

$x^3 \rightarrow x^3 = x^3$

$x^2 \rightarrow -px^2 + kx^2 = 0x^2 \rightarrow -p + k = 0$

$x \rightarrow x - kpx = -2x \rightarrow 1 - kp = -2 \rightarrow kp = 3$

Constants$\rightarrow$ $k = -3r$

$-p - 3r = 0$ , $-3rp = 3$ (sub in for k with $-3r$)

$p = -3r \rightarrow -3r(-3r) = 3$

$9r^2 = 3$

$r^2 = \frac{1}{3}$

$r = \frac{\sqrt{3}}{3}$

$p = -3r$

$p = -3\left(\frac{\sqrt{3}}{3}\right) = -\sqrt{3}$

**(a)** $f'(x) = 2x + b$ (use calculus so that the local maximum can be involved)

$2(3) + b = 0$ (first derivative = 0, at a turning point)

$b = -6$

$f(x) = x^2 - 6x + c$

$-1 = 3^2 - 6(3) + c$

$-1 = 9 - 18 + c$

$c = 8$

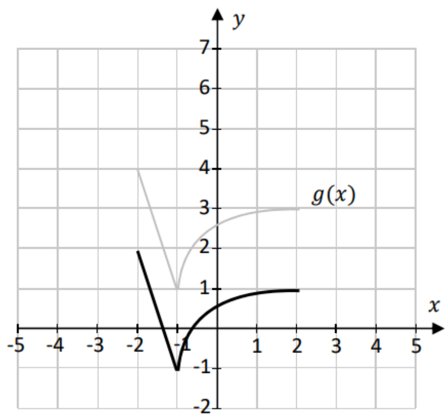
**(b)** $\lim_{n\to\infty}\left(\frac{n}{n+1} + \frac{n+1000}{n} + \left(\frac{1}{3}\right)^n\right)$

$\lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}} + \frac{1+\frac{1000}{n}}{1} + \left(\frac{1}{3}\right)^n\right)$ (for the first two terms, divide the top and bottom by n)

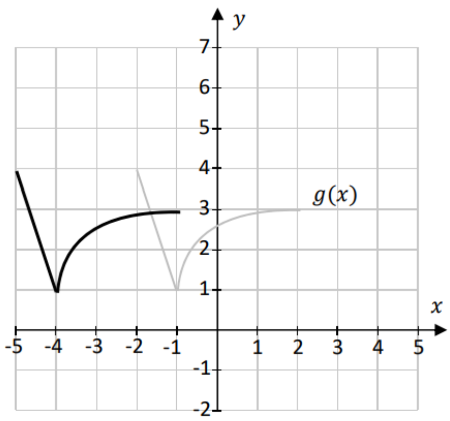
$\frac{1}{1+0} + \frac{1+0}{1} + 0$ (if the limit is infinity, the fraction of $\frac{1}{n}$ will eventually become as close to 0 as possible)

$\frac{1}{1} + \frac{1}{1} = 2$

**(c)** (i)




(ii)



**(a)** Assume $\sqrt{2}$ is rational

Rational numbers can be shown in a fraction form: $\frac{p}{q}$

$(\sqrt{2})^2 = \frac{p^2}{q^2}$

$2q^2 = p^2$ ($p^2$ has to be even as double q)

$p^2$ is even $p$ is even $p = 2k$

$2q^2 = (2k)^2 = 4k^2$

$q^2 = 2k^2$ ($q^2$ has to be even as double k)

$q^2$ is even $q$ is even $q = 2m$

$\sqrt{2} = \frac{p}{q} = \frac{2k}{2m}$

Common factor of 2 (contradiction)

Therefore $\sqrt{2}$ cannot be rational

**(b)** $log_3 t + \frac{log_3 t}{log_3 9} + \frac{log_3 t}{log_3 27} + \frac{log_3 t}{log_3 81} = 10$ (convert all to the base of 3)

$log_3 t\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = 10$

$\frac{25}{12} log_3 t = 10$

$log_3 t = \frac{120}{25}$

$t = 3^{\frac{24}{5}}$

**(c)** (i) $6^x = m$

$log_6 m$ is the 'x' in the above equation, the power applied to 6 to equal m

(ii) $log_6 m > 1$

**(a)** $(1+i)^2+(3-2i)(1+i)+p=0$

$(1+i)(1+i)+3(1+i)-2i(1+i)+p=0$

$1+2i-1+3i-2i+2+p=0$

$5+3i-p=0$

$p=-5-3i$

**(b)** $w=\sqrt{-1+\sqrt{3}i}$

$w=\left(-1+\sqrt{3}i\right)^{\frac{1}{2}}$

$r^2=1^2+\left(\sqrt{3}\right)^2=4 \quad , \quad r=2$

$\theta=tan^{-1}\left(\frac{\sqrt{3}}{1}\right)=\frac{\pi}{3} \quad ,\pi-\frac{\pi}{3}=\frac{2\pi}{3}$

General form: $w^2=2\left(\cos\left(\frac{2\pi}{3}+2\pi n\right)+isin\left(\frac{2\pi}{3}+2\pi n\right)\right)$

$w=\left(2\left(\cos\left(\frac{2\pi}{3}+2\pi n\right)+isin\left(\frac{2\pi}{3}+2\pi n\right)\right)\right)^{\frac{1}{2}}$

$w=\sqrt{2}\left(\cos\left(\frac{\pi}{3}+\pi n\right)+isin\left(\frac{\pi}{3}+\pi n\right)\right)$

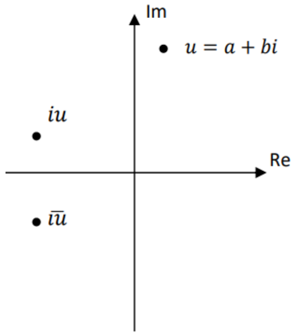
$n=0 \quad w=\sqrt{2}\left(\cos\left(\frac{\pi}{3}+\pi(0)\right)+isin\left(\frac{\pi}{3}+\pi(0)\right)\right)=\frac{\sqrt{2}}{2}+\frac{\sqrt{6}}{2}i$

$n=1 \quad w=\sqrt{2}\left(\cos\left(\frac{\pi}{3}+\pi(1)\right)+isin\left(\frac{\pi}{3}+\pi(1)\right)\right)=-\frac{\sqrt{2}}{2}-\frac{\sqrt{6}}{2}i$

**(c)** (i) $iu = i(a + bi)$ $\overline{iu} = -b - ai$

$-b + ai$

(ii)




(iii) Axial symmetry in a line through the origin with slope -1 or similar

**(a)** Quotient rule: $\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

$u = 1 \quad , \quad v = 5x^2 + 7$

$\frac{du}{dx} = 0 \quad , \quad \frac{dv}{dx} = 10x$

$\frac{(5x^2+7)(0)-1(10x)}{(5x^2+7)^2}$

$-\frac{10x}{5x^2+7}$

**(b)** $g(x) = \left(\tan\left(\frac{x}{2}\right)\right)(lnx)$

Product rule: $u\frac{dv}{dx} + v\frac{du}{dx}$

$u = \tan\left(\frac{x}{2}\right) \quad , \quad v = lnx$

$\frac{du}{dx} = \frac{1}{2}sec^2\left(\frac{x}{2}\right) \quad , \quad \frac{dv}{dx} = \frac{1}{x}$

$g'(x) = \tan\left(\frac{x}{2}\right)\left(\frac{1}{x}\right) + (lnx)\left(\frac{1}{2}sec^2\left(\frac{x}{2}\right)\right)$

$g'\left(\frac{\pi}{2}\right) = \tan\left(\frac{\left(\frac{\pi}{2}\right)}{2}\right)\left(\frac{1}{\left(\frac{\pi}{2}\right)}\right) + \left(\ln\left(\frac{\pi}{2}\right)\right)\left(\frac{1}{2}sec^2\left(\frac{\left(\frac{\pi}{2}\right)}{2}\right)\right)$

$\frac{2}{\pi} + \left(ln\frac{\pi}{2}\right)(1)$

$\frac{2}{\pi} + ln\frac{\pi}{2}$

**(c)** (i) $g\big(f(3)\big) = w$

(ii) **Injective**: no element of C is used more than once

**Not surjective**: range does not equal codomain

**(a)** (i) $x + 4 = x^2 - 2$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3\ ,\ x = -2$

(ii) $x + 4 - (x^2 - 2)$ $\qquad (f(x) - g(x))$

$\int_{-1}^{2}(-x^2 + x + 6)\, dx$

$\Big\{_{-1}^{2} \left(-\frac{x^3}{3} + \frac{x^2}{2} + 6x\right)$

$\left(-\frac{(2)^3}{3} + \frac{(2)^2}{2} + 6(2)\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 6(-1)\right)$

$\frac{33}{2}$ units$^2$

**(b)** $\int_0^b be^{bx} dx = e$

$\Big\{_0^b e^{bx} = e$

$e^{b(b)} - e^{b(0)} = e$

$e^{b^2} - 1 = e$

$e^{b^2} = e + 1$

$b^2 = \ln{(e + 1)}$

$b = \sqrt{\ln{(e + 1)}} = 1.15$

**(a)** $v(0) = \frac{2}{3}(0)^3 - 6(0)^2 + 13(0) + 109 = 109$ km/hr

**(b)** $v'(t) = 2t^2 - 12t + 13$

$v'(5) = 2(5)^2 - 12(5) + 13 = 3$ km/hr/min

**(c)** $v'(t) = 2t^2 - 12t + 13 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(13)}}{2(2)}$

$\frac{12 \pm \sqrt{144 - 104}}{4}$

$\frac{12 \pm \sqrt{40}}{4}$

$t = 1.42 \quad or \quad$ ~~$t = 4.58$~~ (has to be less than 4 minutes)

$t = 1.42$ minutes

**(d)** $\int_0^5 \frac{2}{3}t^3 - 6t^2 + 13t + 109\, dx$

$\frac{1}{5}\left\{_0^5 \left(\frac{t^4}{6} - 2t^3 + \frac{13t^2}{2} + 109t\right)\right.$

$\frac{1}{5}\left[\left(\frac{(5)^4}{6} - 2(5)^3 + \frac{13(5)^2}{2} + 109(5)\right) - \left(\frac{(0)^4}{6} - 2(0)^3 + \frac{13(0)^2}{2} + 109(0)\right)\right] = 112.33$ km/hr

**(e)** Answer B

$v'(1) > 0$ Function is increasing with a positive slope

$v''(1) < 0$ Rate of increase is slowing

**(f)** $\frac{10km}{100} = 0.1$ hours

$0.1 \times 60 = 6$ minutes

**(g)** $\frac{10}{100} \times 60 = 6$ minutes for total journey

2 mins @ $120$ km/hr

$6 - 2 = 4$ remaining minutes

2 minutes at $20$ km/hr above average

$4$ remaining minutes at ? below average

Twice as long as above therefore half the amount below the average $= 10$ km/hr

$100$ km/hr $-10$ km/hr $= 90$ km/hr for last 6 km (4 minutes)

Highest speed: $120$ km/hr

Average speed: $90$ km/hr

Therefore lowest speed: 60 km/hr

Decrease across 4 minutes: $120$ km/hr $-$ $60$ km/hr $= 60$ km/hr

Deacceleration $= \frac{60}{4} = 15$ km/hr/min

**(a)** $P(1+i)^t = F$

$3{,}000(1.024)^5 = €3{,}377.70$

**(b)** (i) It is the amount that should be invested today to amount to €1000 in 1 years' time at the particular interest rate.

(ii) $P(1+i)^t = F$

$P(1.024)^6 = 4{,}000$

$P = \frac{4{,}000}{1.024^6} = €3{,}469.45$

**(c)** $(1 + yearly\ rate)^1 = (1 + monthly\ rate)^{12}$

$(1.024)^1 = (1+i)^{12}$

$1.024^{\frac{1}{12}} = 1 + i$

$i = 1.005947 - 1 = 0.005947$

$i = 0.59\%$

**(d)** (i) $A(1.0011^{36} + 1.0011^{35} + \cdots + 1.0011^2 + 1.0011^1)$

(ii) $S_n = \frac{a(1-r^n)}{1-r}$

$a = 1.0011 \quad , \quad r = \frac{1.0011^2}{1.0011^1} = 1.0011 \quad , \quad n = 36$

$A\frac{1.0011\left(1-(1.0011)^{36}\right)}{1-1.0011} = 12{,}000$

$A(36.742) = 12{,}000$

$A = €326.60$

**(e)** $0.52(11) + (x - 5)(0.15) + x(0.33) = 13.85$

$5.72 + 0.15x - 0.75 + 0.33x = 13.85$

$0.48x = 13.85 - 5.72 + 0.75$

$0.48x = 8.88$

$x = €18.50$

**(f)** Margin: $18\%$ of selling price

Cost price: $100\% - 18\% = 82\%$

Profit: $18\%$

Mark up: $\frac{18}{82} \times 100 = 22\%$

**(a)** (i) $2^0,\ 2^1,\ 2^2,\ 2^3,\ 2^4$

(ii) $3^7$

$3^0,\ 3^1,\ 3^2,\ 3^3,\ 3^4,\ 3^5,\ 3^6,\ 3^7$

$8$

(iii) $2^{10} = 11$ factors , $3^{12} = 13$ factors

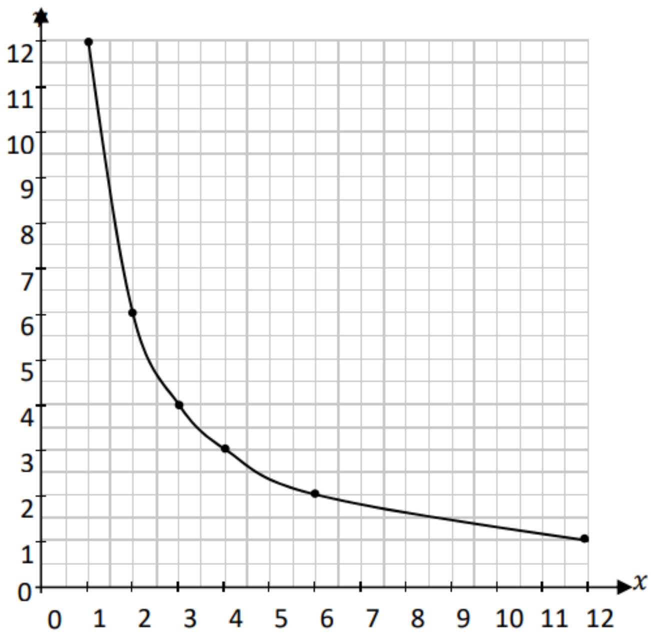
$11 \times 13 = 143$

**(b)** (i)

| $x$ | 1 | 2 | 3 | **4** | 6 | **12** |
|---|---|---|---|---|---|---|
| $y$ | **12** | **6** | 4 | **3** | **2** | 1 |

(ii)

(iii)



**(c)** (i) $y = \frac{12}{x}$

$\frac{dy}{dx} = -12x^{-2}$

$-12p^{-2}$ = slope at $p\ (m)$ , point: $\left(p\,, \frac{12}{p}\right)$

Equation of the line: $y = mx + c$

$\frac{12}{p} = (-12p^{-2})(p) + c$

$\frac{12}{p} = -\frac{12}{p^2}(p) + c$

$\frac{12}{p} = -\frac{12}{p} + c$

$\frac{24}{p} = c$

$y = -\frac{12}{p^2}x + \frac{24}{p}$

(ii) y-intercept: $y = -\frac{12}{p^2}(0) + \frac{24}{p}$ (when $x = 0$)

$y = \frac{24}{p}$ (length of height)

x-intercept: $0 = -\frac{12}{p^2}x + \frac{24}{p}$

$\frac{12}{p^2}x = \frac{24}{p}$

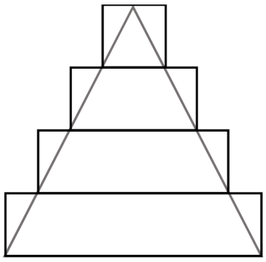
$x = \frac{24}{p}\left(\frac{p^2}{12}\right)$

$x = 2p$ (length of base)

Area of triangle: $\frac{1}{2}(base \times height)$

$\frac{1}{2}\left(2p \times \frac{24}{p}\right) = 24$ units$^2$

**(a)**



**(b)** Height of each rectangle $= \frac{8}{3}$

Width of each rectangle: 2, $\frac{4}{3}$, $\frac{2}{3}$

Area of each rectangle: $\frac{8}{3}(2) + \frac{8}{3}\left(\frac{4}{3}\right) + \frac{8}{3}\left(\frac{2}{3}\right) = \frac{32}{3}$ u$^2$

**(c)** $T_3 = \frac{8}{3}\left(\frac{2}{3} + \frac{4}{3} + \frac{6}{3}\right)$

$T_4 = \frac{8}{4}\left(\frac{2}{4} + \frac{4}{4} + \frac{6}{4} + \frac{8}{4}\right)$

$T_n = \frac{8}{n}\left(\frac{2}{n} + \frac{4}{n} + \cdots + \frac{2n}{n}\right)$

$T_n = \frac{8}{n^2}(2 + 4 + \cdots + 2n)$

$\frac{8}{n^2} \times S_n$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$a = 2 \ , d = 2$

$\frac{n}{2}(4 + (n-1)2)$

$\frac{n}{2}(4 + 2n - 2)$

$S_n = \frac{n}{2}(2n - 2) = n(n+1)$

$\frac{8}{n^2} \times n(n+1)$

$\frac{8}{n}(n+1)$

**(d)** Area of triangle: $\frac{1}{2}(2)(8) = 8$

$0.95(8)$

$\frac{8(n-1)}{n} > 8(0.95)$

$\frac{n-1}{n} > 0.95$

$n - 1 > 0.95n$

$0.05n > 1$

$n > 20$

Greater than 95%, $n = 21$

**(e)** (i) $\int_0^h \frac{x^2c^2}{h^2}dx$

$\left\{\begin{matrix} h \\ 0 \end{matrix}\right. \left(\frac{x^3c^2}{3h^2}\right)$

$\frac{h^3c^2}{h^2 3} - \frac{0^3(c^2)}{3h^2}$

$\frac{hc^2}{3}$

(ii) Asked for: $\frac{dSx}{dt}$ when $2x = h$

Known: $\frac{dx}{dt} = 3u/s$

Link: $\frac{dx}{dt} \times \frac{\mathbf{dSx}}{\mathbf{dx}} = \frac{dSx}{dt}$

$S(x) = \frac{x^2c^2}{h^2}$

$\frac{Sx}{dx} = \frac{2xc^2}{h^2}$

$\frac{dx}{dt} \times \frac{\mathbf{dSx}}{\mathbf{dx}}$

$3 \times \frac{2xc^2}{h^2}$

$3 \times \frac{2xc^2}{h^2}$ when $2x = h$

$3\left(\frac{hc^2}{h^2}\right) = \frac{3c^2}{h}$