(a) Chance of getting 6 → $\frac{5}{12}$

Chance of getting 9 → $\frac{3}{12}$

$\frac{5}{12} \times \frac{3}{12} \times \frac{5}{12} = 0.0434$

(b) Bernoulli Trials: $3rd$ time getting 9 on $8th$ time:

2 successes in first 7 spins then another success

Probability of success: $\frac{1}{4}$, probability of failure: $\frac{3}{4}$

$\binom{7}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^{7-2} \times \frac{1}{4} = 0.0779$

(c)

$P(0,0)$	$\frac{4}{12} \times \frac{4}{12}$	$P(0\text{ first})$	$\frac{4}{12}$
$P(0,6)$	$\frac{4}{12} \times \frac{5}{12}$		
$P(0,9)$	$\frac{4}{12} \times \frac{3}{12}$		
$P(6,0)$	$\frac{5}{12} \times \frac{4}{12}$	$P(6\text{ first})$	$\frac{5}{12}$
$P(6,6)$	$\frac{5}{12} \times \frac{5}{12}$		
$P(6,9)$	$\frac{5}{12} \times \frac{3}{12}$		
$P(9,0)$	$\frac{3}{12} \times \frac{4}{12}$	$P(9,0)$	$\frac{1}{12}$
$P(9,6)$	$\frac{3}{12} \times \frac{5}{12}$	$P(9,6)$	$\frac{5}{48}$
TOTAL	$= \frac{15}{16}$		$= \frac{15}{16}$

$\frac{15}{16} = 0.9375$

(a) $\cos(A - B) = cosAcosB + sinAsinB$

Replace A with $90 - A$

$\cos(90 - A - B) = \cos(90 - A)cosB + \sin(90 - A)sinB$

$\cos\big(90 - (A + B)\big) = sinAcosB + cosAsinB$ as $sinA = \cos(90 - A)$

$\sin(A + B) = sinAcosB + cosAsinB$ QED

(b) $\sin(30 + 45) = \sin(30)\cos(45) + \cos(30)\sin(45)$

$= \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right) + \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} \rightarrow \frac{1+\sqrt{3}}{2\sqrt{2}}$

$\frac{1+\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}+\sqrt{2}\sqrt{3}}{4}$

$\frac{\sqrt{2}+\sqrt{6}}{4}$

(c) $\sin(t) = \sin(2t)$

$sin2A = 2sinAcosA$

$\sin(t) = 2\sin(t)\cos(t)$

$\sin(t) - 2\sin(t)\cos(t) = 0$

$\sin(t)\,(1 - 2\cos(t)) = 0$

$\sin(t) = 0\ ,\ 1 - 2\cos(t) = 0$

$\sin(t) = 0 \rightarrow t = 0°, 180°, 360°$

$\cos(t) = \frac{1}{2}$

$t = 60°, 300°$

$t = 0°, 60°, 180°, 300°, 360°$

(a) Translate the points: $(0, 11) \rightarrow (0, 0)$ $\quad y: -11$

$(4, 6) \rightarrow (4, -5)$

$(-3, -1) \rightarrow (-3, -12)$

Area: $\frac{1}{2}|x_1y_2 - x_2y_1|$

$\frac{1}{2}|(4)(-12) - (-3)(-5)|$

31.5 units^2

(b) (i) Midpoint formula: $\left(\frac{x_2-x_1}{2},\frac{y_2-y_1}{2}\right)$

$\left(\frac{5+(-1)}{2},\frac{k+l}{2}\right)$

$\left(\frac{4}{2},\frac{k+l}{2}\right)$

$\left(2,\frac{k+l}{2}\right)$

(ii) Slope of bisector: $-\frac{a}{b}=-\frac{3}{2}$, therefore slope of $|AB|$: $-\frac{3}{2}\times m=-1$

$m=\frac{2}{3}$

Equation of the line formula $y-y_1=m(x-x_1)$

Equation of $|AB|$: $y-k=\frac{2}{3}(x-(-1))$

Sub in other point: $l-k=\frac{2}{3}(5+1)$

$-k+l=4$

Sub in midpoint to equation of perpendicular bisector: $3(2)+2\left(\frac{k+l}{2}\right)-14=0$

$6+k+l-14=0$

$k+l=8$

Now simultaneous equations: $k+l=8$

$-k+l=4$

$2l=12$

$l=6$

Sub back in to find k: $k+(6)=8$

$k=2$

(a) (i) Centre: $(h, -3)$

Radius: $\sqrt{12}$

(ii) Perpendicular distance between a line and point formula: $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$\frac{|1(h)-4(-3)+7|}{\sqrt{(1)^2+(-4)^2}}$

$\frac{|h+19|}{\sqrt{17}} = 5$

$\frac{(h+19)^2}{17} = 25$

$h^2 + 38h + 361 = 425$

$h^2 + 38h - 64 = 0$

$h = 5\sqrt{17} - 19 \quad or \quad h = -5\sqrt{17} - 19$

(b) Sub in each point: $x^2 + y^2 + 2gx + 2fy + c = 0$

$8^2 + 1^2 + 2(8)g + 2(1)f + c = 0 \quad \rightarrow \quad 65 + 16g + 2f + c = 0$

$a^2 + 3^2 + 2ag + 2(3)f + c = 0 \quad \rightarrow \quad a^2 + 9 + 2ag + 6f + c = 0$

$a^2 + (-5)^2 + 2ag + 2(-5)f + c = 0 \quad \rightarrow \quad a^2 + 25 + 2ag - 10f + c = 0$

$a^2 + 9 + 2ag + 6f + c = 0$

$-a^2 - 25 - 2ag + 10f - c = 0$

$-16 + 16f = 0$

$f = 1$

$65 + 16g + 2(1) + c = 0$

$16g + c = -67$

$c = -67 - 16g$

$\sqrt{g^2 + f^2 - c} = radius$

$\sqrt{g^2 + 1 - c} = \sqrt{20}$

$g^2 - c = 19$

$g^2 + 16g + 67 = 19$

$g^2 + 16g + 48 = 0$

$(g + 4)(g + 12) = 0$

$g = -4$

$(-4)^2 - c = 19$

$16 - c = 19$

$c = -3$

$x^2 + y^2 - 8x + 2y - 3 = 0$

(a) (i) Mean: $\frac{0+3+2+2+4+5+1}{7} = \frac{17}{7} = 2.42$

Standard deviation (from calculator): 1.6

(ii) correlation coefficient: $r = -0.762$

(iii) If the number of red cubes increases then there will be less green cubes

(b)

3 faces:	8
2 faces:	24
1 face:	22
no faces:	6

(a) Answer: False

Justification: 2 angles can be equal in size without being vertically opposite e.g. two of the same angles in an isosceles triangle.

(b) (i) $|\angle EHD| = |\angle DBC| = \theta$ ... alternate angles $|\angle EFD| = |\angle EHD|$ ... both $= \theta$

$|\angle FED| = |\angle HED|$ ...rectangle & straight angle $|\angle FDE| = |\angle HDE|$... angles in tri. sum to 180

$|ED| = |ED|$ ... common side Conclusion: So $FED \equiv HED$ ... by ASA $|FE| = |EH|$

(ii) $|EH| + |FE| = |AB|$, and $|EH| = |FE|$, therefore $|AB| = 2|FE|$

$2|FE| = 20$

$|FE| = 10$

$|FG| = 50 - 10 = 40$

$tan\theta = \frac{opposite}{adjacent} = \frac{|AG|}{|FG|} = \frac{90}{40}$

$\theta = tan^{-1}\left(\frac{9}{4}\right) = 66°$

(a) $|AC|^2 + 9^2 = 70^2$

$|AC|^2 = 4900 - 81$

$|AC| = \sqrt{4819}$

Gradient: $\frac{|BC|}{|AC|} = \frac{9}{\sqrt{4819}} \times 100 = 13\%$

(b) Find $|RO|$

Find other angle in triangle ROP: $180 - 88 - 87 = 5°$

Sine rule: $\frac{a}{SinA} = \frac{b}{SinB}$

$\frac{x}{Sin87} = \frac{20}{Sin5}$

$x = \left(\frac{20}{Sin5}\right)(Sin87) = 229.16$ m

SohCahToa: $Tan\theta = \frac{opposite}{adjacent}$

$Tan17 = \frac{y}{229.16}$

$y = 229.16(Tan17) = 70$ m

(c) $a = 1.6 \quad , \quad b = 2.4$

(d) If $V'(t) > 0$ then the volume of air is increasing so she is breathing in. If $V'(t) < 0$ then the volume of air is decreasing so she is breathing out.

(e) (i) $V(0.5) = 2 - 0.4\cos\left(\frac{\pi}{2}(0.5)\right) = 1.717$ litres

(ii) $V'(t) = -0.4\left(-\sin\left(\frac{\pi}{2}t\right)\right)\left(\frac{\pi}{2}\right)$

$V'(0.5) = 0.2\pi\sin\left(\frac{\pi}{2}(0.5)\right) = 0.444$ litres/second

(f) General equation : $b + acos(ct)$

Max: 3.6

Min: 1.3

Midline: $b = 2.45$

Amplitude: $a = 1.15$

Period: $2\ seconds$

$c = \frac{2\pi}{period} = \frac{2\pi}{2} = \pi$

Negative 'cos' graph

$2.45 - 1.15\cos(\pi t)$

(a) $\frac{x-\mu}{\sigma}$

$\frac{3.5-3.87}{0.36} = -1.03$

$P(1.03) = 0.8485$

$1 - 0.8485 = 0.1515$

(b) (i) 95% confidence interval: $\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$

$3.74 + 1.96\left(\frac{0.36}{\sqrt{64}}\right) = 3.8282$

$3.74 - 1.96\left(\frac{0.36}{\sqrt{64}}\right) = 3.6518$

$3.6518 < \mu < 3.8282$

(ii) $H_0: \mu = 3.87$

$H_A: \mu \neq 3.87$

Reject the null hypothesis

3.87 is not within the confidence interval: $3.6518 < \mu < 3.8282$

(c) $\bar{x} = 0.35$

Confidence interval: $0.265 < \mu < 0.435$

$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$0.35 + 1.96\sqrt{\frac{0.35(1-0.35)}{n}} = 0.435$

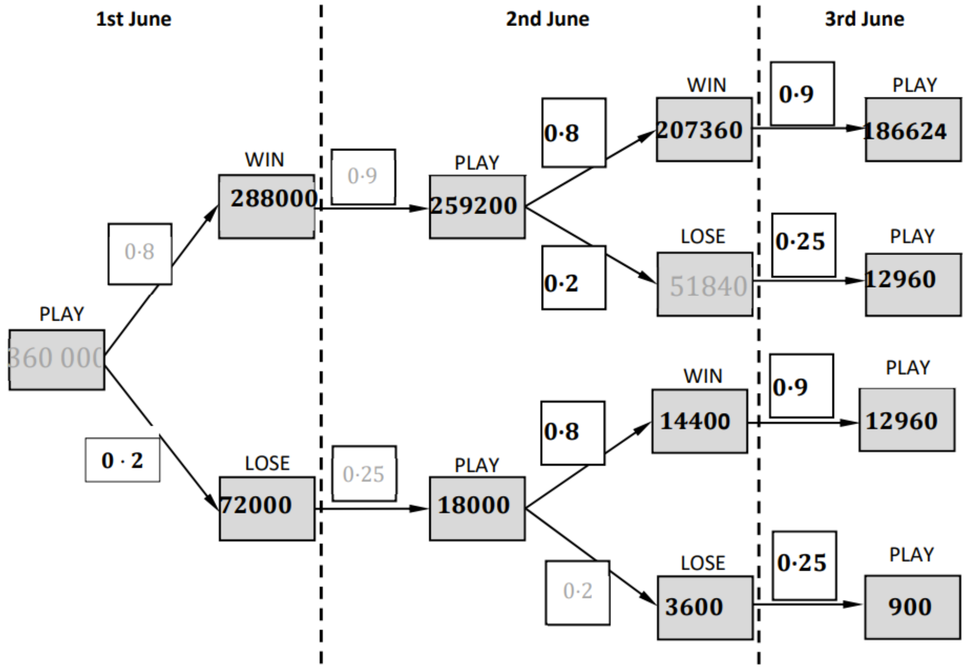
$1.96\sqrt{\frac{0.2275}{n}} = 0.085$

$\sqrt{\frac{0.2275}{n}} = \frac{0.085}{1.96}$

$\frac{0.2275}{n} = \left(\frac{0.085}{1.96}\right)^2$

$n\left(\frac{0.085}{1.96}\right)^2 = 0.2275 \qquad n = \frac{0.2275}{\left(\frac{0.085}{1.96}\right)^2} = 121$

(d) (i)



(ii) Total players: $186624 + 12960 + 12960 + 900 = 213444$

Lost on the 1st/2nd: $12{,}960 + 900 + 12{,}960 = 26{,}820$

$$\frac{26{,}820}{213{,}444} = \frac{745}{5{,}929}$$

(a) (i) $l = \sqrt{140} = 11.83$

(ii) Area of triangle: $\frac{1}{2}abSinC$

$\frac{1}{2}(x)(x)Sin(60)$

$\frac{x^2(\sqrt{3})}{4} = \frac{140}{6}$

$x^2 = \frac{140}{6}\left(\frac{4}{\sqrt{3}}\right) = \frac{280}{3\sqrt{3}}$

$x = \sqrt{\frac{280}{3\sqrt{3}}} = 7.3$ cm

(b) (i) Cosine rule: $a^2 = b^2 + c^2 - 2bcCos\alpha$

$4^2 = 6^2 + 8^2 - 2(6)(8)Cos\alpha$

$-84 = -96cos\alpha$

$Cos\alpha = \frac{84}{96}$

$\alpha = cos^{-1}\left(\frac{84}{96}\right)$

$\alpha = cos^{-1}\left(\frac{7}{8}\right)$

(ii) $\alpha = cos^{-1}\left(\frac{7}{8}\right) = 28.955$

$|AC|$: $cos\alpha = \frac{adjacent}{hypotenuse}$

$\cos(28.955) = \frac{8}{|AC|}$

$|AC| = \frac{8}{\cos(28.955)} = \frac{64}{7}$

Area of a triangle: $\frac{1}{2}abSinC \rightarrow \frac{1}{2}(8)\left(\frac{64}{7}\right)Sin(28.955) = 35.41$ cm^2

(c) (i) Radius$= 1$

x-value: $cos45 = \frac{x}{1}$

$x = \frac{1}{\sqrt{2}}$

$x = -\frac{1}{\sqrt{2}}$ (2nd quadrant)

y-value : $sin45 = \frac{y}{1}$

$y = \frac{1}{\sqrt{2}}$

$\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(ii) Find the equations of the line for the tangents at P and Q

Slope formula: $\frac{y_2 - y_1}{x_2 - x_1}$

$Q = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, slope of line perpendicular to tangent: $\frac{\frac{1}{\sqrt{2}} - 0}{-\frac{1}{\sqrt{2}} - 0} = -1$

Slope of tangent at Q: $m \times -1 = -1 \quad \rightarrow \quad m = 1$

Equation of the line formula: $y - y_1 = m(x - x_1)$

$y - \frac{1}{\sqrt{2}} = 1\left(x - \left(-\frac{1}{\sqrt{2}}\right)\right)$

$y - \frac{1}{\sqrt{2}} = x + \frac{1}{\sqrt{2}}$

$y = x + \sqrt{2}$

Equation of other tangent at P: $x = -1$ (sub into equation for tangent at Q)

$y = -1 + \sqrt{2}$

Centre: $(1, -1 + \sqrt{2})$

Radius: $-1 + \sqrt{2}$

(a) (i) $\frac{12}{15} = \frac{x}{x+10}$

$12x + 120 = 15x$

$120 = 3x$

$x = 40$ cm QED

(ii) Surface area formula: πrl

S.A. of large cone: $\pi(15)(50) = 750\pi$

S.A. of small cone: $\pi(12)(40) = 480\pi$

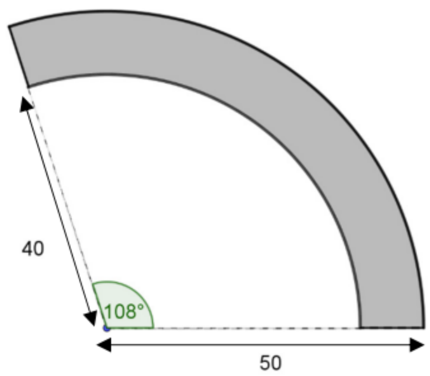
$750\pi - 480\pi = 270\pi$ (S.A. of basin without the bottom)

Area of bottom: $\pi r^2 \rightarrow \pi(12)^2 = 144\pi$

$270\pi + 144\pi = 414\pi = 1{,}300.6$ cm^2

(iii) Angle: $\frac{\theta}{360} 2\pi(40) = 2\pi(12)$

$\theta = 360\left(\frac{3}{10}\right) = 108°$



(b) (i) $9 \times 8 \times 7 \times 6 = 3{,}024$

(ii) $8 \times 7 \times 6 \times 5 = 1{,}680$

$3{,}024 - 1{,}680 = 1{,}344$

(iii) $1 + 2 + 3 = 6$

$1 + 2 + 4 = 7$

$1 + 2 + 5 = 8$

$1 + 2 + 6 = 9$

$1 + 3 + 4 = 8$

$1 + 3 + 5 = 9$ 7 possible combinations with 6 arrnagements for each

$2 + 3 + 4 = 9$ $7 \times 6 = 42$